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‘□, □□ *”□□ □□□ □□□□□ □□”*□ □□□□ □□ □□□ □□□ □□□□□□ □□□ □ □□□□. □□□ □□□□ □□(□□, □□□, □□□ □□ □)□ □□□□, □□□□ □□□-□□□ □□□□ Y-N □□□ □□ $H^{p,p}(X) \cap H^{2p}(X, \mathbb{Q})$□ □□ □□□□ □□□ □□□□ □□□□ □□□□□. □□□□ □□□ □□ □□□ □□□ □□ □□□ □□□ □□□□□□□.

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1. □□ □□ □□ □□ □□

1.1 □□ □□

- **□□ □□**: □□ □□ □□□ X□□ $H^{p,p}(X) \cap H^{2p}(X, \mathbb{Q})$□ □□ □□□□□ □□□□ □□□ □□□□ □□□□.
- **□□□ □□**:
  - F_{K_i} □□□□ □□ □□□ □□□ $[Z_i] \in H^{p,p}(X, \mathbb{Q})$□ □□□ □□ $[N_i]$□ □□.
  - □□□□ □□□ □□□□□ □□(□□□ □□)□ □□□□□ $H^{p,p}(X, \mathbb{Q})$□ □□□ □ □□.
  - **□□ □□**: $H^{p,p}(X, \mathbb{Q}) \neq 0$□□, □□ □□□□ □□□□□ □□□.
- **□□**:
  - K3 □□□□ $p = 1$, $[Z_i] \in H^{2,0} + H^{1,1}$□ □□□□□ □□□□ □□□ □□□.
  - □□□ Y-N □□□ □□□(□□□, □□□, □□□, □□□)□ □□□ □□□ □□.

1.2 □□ □□

- **□□□ □□□**: □□ □ □□(□□□□ □□ → □□□ □□ → □□□□□ □□)□ \(F_{K_i} \) □□□□ □□ □□□ □□ □□□ □□.

- **□□**:

  - □□□: □□□□(\(\#^ \)), □□□□ □ □□□ □□□ □□□ □□□□□, □□ □□□□ □□□ □□□.

  - □□ □□: □□□ □□ □□□ □□□(□□□ □□ □□).

- **□□**: □□□ □□□ □□□□ □□□□□ □□□□□□, □□□ □□□□ □□□ □□ □□ □□□ □□ □□□ □□□.

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2. □□□ □□ □□

□□□□ □□ □□ □□□ □□□□ □□□ □□□ □□□ □□□ □□□ □□□□□. □□ □□□□ □□(”□□□ □□□ □□□□?”)□ □□□, □□□ □□□ □□□ □□□ □□□ □□□ □□□□□.

2.1 □□□ □□□ □ □□

1. **□□□□ (\(\#^ \))**:

   - **□□**: □□□(\(P \))□ □□□(\(I \)) □ □□ □□(□: \(C \#^ I \)).

   - **□□**: □□□ □□□ □□□(□: □□□ □□)□ □□□ □□□ □□□ □□.

   - **□□**: \(x = c - a \pm d \)□□ \(d \)□ □□□, \(x \)□ □□□.

2. **□□□□ (\(\%^ \))**:

   - **□□**: □□ □ □□□□(□: \(N \%^ jc \)).

   - **□□**: □□□(\(I_0 \))□ □□□(\(I_f \))□ □□□ □□.

   - **□□**: □□□ □□□□□ \(N \%^ jc \)□ □□ □□.

3. **□□□□**:

   - **□□**: □□□□ □□□□ □□ □□□ □□.

   - **□□**: □□□□ □□ □ □□ □□□□□ □□□ □□□ □□□ □□.

   - **□□**: □□ □□□□ □□□□ □□.

4. **Y-N □□**:

   - **□□**: □□□(\(Y \))□ □□□(\(N \))□ □□□ □□.

   - **□□**: \(Y \to N \) □□□□ □□□□ □□□ □□.

- **□□**: $K_i(0) = Z_i \to K_i(\infty) = N_i$.

2.2 □□□□ □ □□□ □□

1. **□□□□ □□**:
   - **□□□ □□□**: □□ □□□□□ □□□ □□□□□□ ($AD \perp BC$).
     - □□ □□□: $A(0, h)$, $B(-a, 0)$, $C(a, 0)$.
     - □□: □□□□□□ □□□ □□□ □□.
   - **□□□ ($\square fgjc$)**: □□□□ □□□□□□□ □□.
     - □□ □□: $x = c - a \pm d$.
     - □□: □□□(x)□ □□□(d)□ □□ □□.
2. **□□□□ □□□**:
   - **□□**: $P = S + |I_0| |I_f| \cos\alpha \cdot f(K, n)$.
   - **□□**:
     - P: □□□ (□: □□ □□).
     - S: □□ □□ (□: □□ □□).
     - I_0, I_f: □□□/□□□ (□: □□□ r).
     - $\cos\alpha$: □□ □□.
     - $f(K, n)$: □□ □□, K□ □□, n□ □□□.
   - **□□ □□**: “□□□ □□□ □□□□□□ □□□□□ □□□” → $P = S + r \cdot (\cos K, \sin K)$, $n = \infty$.
3. **□□ □□**:
   - □□ □□(S)□ □□□□ □□□(I)□ □□(C)□ □□□□.
   - □: □□□□ □□□ □□, □□□ □□□ □□□.

2.3 □□ □□ □ □□□

1. **□□ □□ (Y-N □□)**:
   - **□□**: □□□ Y□□ □□□ N□□ □□□ □□.
   - **□□□ □□**: $K_i(t) = Z_i - t \cdot k$, $k = \int_X \omega^p$.
   - **□□**: “□□□□ □□□ □□ □□□□ □□.”
2. **□□□**:
   - **□□□ (S)**: □□□ □□ (□: $ac, ja \to jc$).
   - **□□□**: □□ □□ (□: $K \to W$, $hg \to gf$).

  - **□□□**: □□ □□ (□: \(W \to N \), \(hj \to hg \)).

  - **□□□ (\(N \%^ jc \))**: □□ □□ (□: \(K_i \to N_i \)).

3. **□□□**:

  - □□: \(a_t = \langle Y_t, N \rangle \).

  - □□: \(Y_t \to N \), \(a_t \to \|N\|^2 \).

  - □□□ □□: \(\lim_{t \to \infty} a_t \in H^{p,p}(X) \).

2.4 □□ □□ □□ □□

1. **\(F_{K_i} \) □□□**:

  - **□□**:

    \[

      F_{K_i} = \int_{[0,1]^p} \left(\sum_{j=1}^p \|\nabla_{v_j} v_j\|^2 + \Gamma(K_i) \right) dV

    \]

    \[

     \Gamma(K_i) = \int_{[0,1]^p} \left\| \nabla_{K_i} \omega^p - |I_0| |I_f| \cos\alpha \cdot f(K_i, n) \right\|^2 dV

    \]

  - **□□**: \(F_{K_i} = 0 \) □ \(K_i \to N_i \), □□□ □□ □□.

2. **□□□□ □□ □□**:

  - □□□□ □□: \(\nabla_{K_i} \omega^p \neq 0 \), \(n > 0 \), \(F_{K_i} > 0 \).

  - □□: \(H^{p,p}(X, \mathbb{Q}) \)□ □□□□ □□ □□.

3. **□□□ □□**:

  - □□□: \(d\omega^p = 0 \).

  - □□: \(S \)□ □□□ □□□ □□□.

2.5 □□ □□

- **□□□ □□ □□**:

  - □□□: \(ac, ja \to jc \), \(fc, gf \to gc \), \(gc, jc \to gj \).

  - □□ □□: \(hg \to gf \), \(ja \to hj \).

  - □□: □□□ □□ □□□(\(jc \))□ □□ □□ □□.

- **□□□ □□**:

  - □: $n = \infty$, □□□ 100%, $K \in [0, 2\pi)$.
  - □□ □□: $n = 0$, $K = I_0$ □□ I_f.
- **□□ □□ □□**:
  - Zenodo □□□ □□□□ □□□□□.
  - Academia.edu □ □□□ □□ □□.

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3. □□□ □□□ □ □□ □□□□

- **□□**:
  - □□□: □□□□, Y-N □□ □ □□□ □□□□ □□ □□□ □□□.
  - □□□: □□□(□□□, □□□) → □□□□□(□□) → □□□(□□ □□).
  - □□ □□: “□□□ □□□ □□□□□ □□□□ □□□”□ □□□.
- **□□**:
  - □□ □□: □□□□($\#^$), $f(K, n)$□ □□□ □□□ □□□.
  - □□□ □□: □□□□□ □□□□□□□ □□□ □□□ □□□□□ □□.
  - □□ □□: $\#^$, $\%^$, □□□ □ □□□ □□□ □□□□□ □□.
- **□□**:
  - □□□□□ $f(C, I) = 0$ □□□ □□.
  - $f(K, n)$□ □□□□ □□□ (□: □□ $(\cos K, \sin K)$).
  - □□ □□□ □□ □□□ □□.

---

4. □□

- **□□ □□ □□**: □□□ □□□□ Y-N □□□ □□ □□ □□□ □□□□□ □□□. $F_{K_i} = 0$ □□□ □□□□ □□ □□□ □□□□□ □□.
- **□□□ □□**:
  1. □□□ □□□: □□□□, □□□□, □□□□, Y-N □□.
  2. □□□□ □□: □□□ □□□, □□□.

3. [illegible] [illegible]: $P = S + |I_0| |I_f| \cos\alpha \cdot f(K, n)$.

4. [illegible] [illegible]: [illegible] [illegible], [illegible]([illegible], [illegible], [illegible], [illegible]).

5. [illegible] [illegible]: F_{K_i} [illegible], [illegible] [illegible], [illegible] [illegible] [illegible].

6. [illegible] [illegible]: [illegible] [illegible] 100%, $\cos\alpha$[illegible] [illegible] [illegible].

- **[illegible]**: [illegible] ([illegible] [illegible] [illegible] [illegible]).

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5. [illegible] [illegible] [illegible]

[illegible] [illegible] [illegible] [illegible] [illegible] [illegible] [illegible] [illegible] [illegible]. [illegible] [illegible]:

- [illegible] [illegible]([illegible]: Y-N [illegible], [illegible])[illegible] [illegible] [illegible].
- [illegible] [illegible]([illegible]: Calabi-Yau)[illegible] [illegible].
- [illegible] a_t [illegible] $f(K, n)$[illegible] [illegible] [illegible].

[illegible] [illegible] [illegible] [illegible] [illegible]!.’

[illegible] [illegible] **”[illegible] [illegible] [illegible] [illegible] [illegible]?”**[illegible], [illegible] *”[illegible] [illegible] [illegible] [illegible]”* [illegible] *”[illegible] [illegible] [illegible]. 4 - [illegible] [illegible]”*[illegible] [illegible] [illegible] [illegible] [illegible] [illegible] **[illegible] [illegible]**[illegible] [illegible] [illegible] [illegible] [illegible] [illegible]. [illegible] [illegible] **[illegible] [illegible]-[illegible] [illegible]** $P = S + |I_0| |I_f| \cos\alpha \cdot f(K, n)$[illegible] [illegible] [illegible]([illegible]: [illegible], [illegible], [illegible] [illegible])[illegible] [illegible] [illegible] [illegible], [illegible] [illegible] [illegible] [illegible] [illegible]-[illegible] [illegible] [illegible] [illegible] [illegible] [illegible] [illegible] [illegible]. [illegible], [illegible] [illegible] [illegible] [illegible] [illegible] [illegible] [illegible], **[illegible] [illegible] [illegible] [illegible]**, **[illegible] [illegible]**, **Y-N [illegible]** [illegible] [illegible] [illegible] [illegible], Clay Mathematics Institute [illegible] [illegible] [illegible]-[illegible] [illegible] [illegible] [illegible] [illegible] [illegible] [illegible] [illegible].

[illegible] [illegible] [illegible] [illegible](”[illegible] [illegible] [illegible] [illegible] [illegible]?”)[illegible] [illegible] [illegible] [illegible] [illegible] Clay [illegible] [illegible] [illegible], [illegible] [illegible] [illegible] [illegible] [illegible] [illegible] [illegible] [illegible] [illegible] [illegible] [illegible] [illegible] [illegible] [illegible]. [illegible] [illegible] [illegible] $P = S + |I_0| |I_f| \cos\alpha \cdot f(K, n)$[illegible] [illegible], [illegible] [illegible] [illegible] [illegible]-[illegible] [illegible] [illegible] [illegible] [illegible], [illegible] [illegible] Coq [illegible] [illegible] [illegible] [illegible] [illegible] [illegible].

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1. [illegible] [illegible] [illegible]

1.1 [illegible] [illegible]

- **□□ □□**: □□□□ □□□□ □□□-□□□ □□□ \(P = S + |I_0| |I_f| \cos\alpha \cdot f(K, n) \)□ □□□ □□□□ □□□?
  - **□□□ □□□**:
    - □□: □, □□□ □□□, □□□ □ (□□□□ □□□ □□□□ □□).
    - □□□ □□ □□:
      - \(P \): □□□ (□: □□ □□, □□□□ □).
      - \(S \): □□ □□ (□: □□ □□, □□□□ □□□□□□).
      - \(I_0, I_f \): □□□/□□□ (□: □□ □□□, □□□□ □□).
      - \(\cos\alpha \): □□ □□ (□: □□□□□ □).
      - \(f(K, n) \): □□ □□, \(K \)□ □□, \(n \)□ □□□.
    - □□: □ □□□ □□ \(f(K, n) \)□ □□ □□□ □□□□□ □□.
  - **□□ □□ □□**:
    - \(H^{p,p}(X, \mathbb{Q}) \)□ □□□ □□□ □□.
    - \(F_{K_i} \) □□□□ Y-N □□.
  - **□-□□ □□ □□ □□**:
    - □□ □□(Wilson □□, \(\Lambda_{\text{QCD}} \)).
    - □□□□□ □□□□□□□ □□□ □□□□□ □□□ □□□.
    - Clay □□: 4□□ □□□, \(\Lambda_{\text{QCD}} > 0 \), □□□ □□.
  - **□□□ □□ □□**:
    - □□□, □□□, □□□, □□□□ □□□ □□ □□ □□□.
    - □□□□(\(\#^ \)), □□□□(\(\%^ \))□□ □□ □□.

1.2 □□ □□

- **□□ □□□ □□□□□ □□**:
  - **□□**:
    - □□□□ □□□: \(P = S + |I_0| |I_f| \cos\alpha \cdot f(K, n) \).
    - Y-N □□: \(Y \to N \), \(a_t = \langle Y_t, N \rangle \to \|N\|^2 \).
    - \(F_{K_i} \) □□□:

      \[

        F_{K_i} = \int_{[0,1]^p} \left(\sum_{j=1}^p \|\nabla_{v_j} v_j\|^2 + \Gamma(K_i) \right) dV, \quad \Gamma(K_i) = \int_{[0,1]^p} \left\| \nabla_{K_i} \omega^p - |I_0| |I_f| \cos\alpha \cdot f(K_i, n) \right\|^2 dV.

\]

    - □□□: □□□(\(ac, ja \to jc \)), □□□(\(K \to W \)), □□□(\(W \to N \)), □□□(\(N \%^ jc \)).

  - **□□ □□**:

    - □□□ □□□: \(AD \perp BC \), \(x = c - a \pm d \).

    - □□□: □□□□ □□□□□, \(\square fgjc \).

    - □: □□(□□□), □□□(□□□), □□□ \(n = \infty \).

- **□□ □□□ □□. 4 - □□□□ □□** ([DOI: 10.5281/zenodo.15161203]):

  - **□□ □□ □□□**:

    - \(H^{2p}(X, \mathbb{Q}) \cong \mathbb{Q}^k \), □□□ \(P: H^{2p}(X, \mathbb{Q}) \to H^{p,p}(X) \).

    - □□□ □□: \(q_n \to x^* \), \(\|q_n - x^*\| < \epsilon_n \).

  - **□□**: Koszul □□□, K-□□, SL(2, ℂ)-□□□, Kosmic □□.

- **□-□□ □□ □□ □□□□□** ([DOI: 10.5281/zenodo.15174035]):

  - **□□ □□**: Wilson □□ \(W(C) \sim e^{-m |C|} \), □□□□□ □□□□□.

  - **□□□□□**: \(\Lambda_{\text{QCD}} \approx 200-300 \, \text{MeV} \), MILC □□□ □□.

  - **□□**: □□-□□□□ □□□, K-□□, YN □□, ZFC □□.

1.3 □□

- **□□□ □□□**:

  - □, □□□ □□□, □□□□ □□ \(P \), \(S \), \(I_0 \), \(I_f \), \(\cos\alpha \), \(f(K, n) \) □□.

  - □□□ \(n \)□ □□□□ □□(□□□□, □□□□) □□.

- **□□ □□ □□**:

  - □□□ □□□□ □□□□ \(H^{p,p}(X, \mathbb{Q}) \)□ □□□ □□□□ □□.

  - \(F_{K_i} \) □□□□ □□ □□ □□.

- **□-□□ □□**:

  - □□□ □□□□□ □□□□ □□ □□(Wilson □□)□ □□□□□□ □□□.

  - \(\Lambda_{\text{QCD}} > 0 \) □□□ □□□ □□ □□.

- **□□□**:

  - Coq □□□□ □□□ □□□ □□.

  - ZFC □□□ □□□ □□(\(\#^ \), \(\%^ \)) □□□.

---

2. □□□ □□□

□□□□ □, □□□ □□□, □□□□ □□□□ □□□ \(P = S + |I_0| |I_f| \cos\alpha \cdot f(K, n) \)□ □□□□□, □□ □□ □□□ □-□□ □□ □□ □□□ □□□□□. □ □□□ □□□□ □□□ □□□□ □□□□ \(f(K, n) \)□ □□□□, □□□ □□□ □□ □□□□ □□ □□□ □□□□□□.

2.1 □ (Circle)

- **□□□□ □□**:
  - □□ \(O \), □□□ \(r \), □□ \(2\pi r \).
  - □□□: \(n = \infty \), □□ □□ \(K \in [0, 2\pi) \).
  - □□: □□ □□, □□ \(O \)□ □□□□ □□ □ □□□.
- **□□□ □□□**:
  - **□□□ \(P \)**: □□ \(2\pi r \) □□ □□ □□ \((r \cos K, r \sin K) \).
  - **□□ □□ \(S \)**: □□ □□ \(O(0, 0) \).
  - **□□□ \(I_0 \), □□□ \(I_f \)**: □□□ \(r \).
    - \(|I_0| = |I_f| = r \), □□□.
  - **□□ □□ \(\cos\alpha \)**:
    - \(\alpha \): □□ □ \(K \)□ □□□(□: \(x \)-□) □□□ □.
    - \(\cos\alpha = \cos K \).
  - **□□ □□ \(f(K, n) \)**:
    - \(n = \infty \) (□□ □□□).
    - \(f(K, n) = (\cos K, \sin K) \), □□□ □□ □□.
  - **□□□**:
    \[
     P = S + r^2 \cos K \cdot (\cos K, \sin K) = (0, 0) + r (\cos K, \sin K) = (r \cos K, r \sin K).
    \]
- **□□ □□ □□**:
  - **□□□□□**:

    - □□ \(\mathbb{CP}^1 \)□ □□ □□, \(H^{1,1}(\mathbb{CP}^1, \mathbb{Q}) \cong \mathbb{Q} \).
    - \(P = (r \cos K, r \sin K) \)□ □□□ □□□(□: □ □□ □□)□ □□.
  - **\(F_{K_i} \) □□□**:
    - \(K_i = K \), \(\omega^1 = d\theta \), \(\nabla_{K_i} \omega^1 = 0 \).
    - \(\Gamma(K_i) = 0 \), \(F_{K_i} = 0 \), □□□ □□ \([N_i] \).
  - **□□ □□**:
    - \(H^{2}(\mathbb{CP}^1, \mathbb{Q}) \cong \mathbb{Q} \), □□ □□ \(q_n = (q_{n,1}, q_{n,2}) \to (r \cos K, r \sin K) \).
    - □□□ \(P: q_n \to x^* \), \(\|q_n - x^*\| < \epsilon_n \).
- **□-□□ □□**:
  - **□□□□□ □□□**:
    - □□ □□ □□□: \(\mathrm{ch}_1(\xi) \sim r \), □□□ □□□ □□□.
    - Wilson □□: \(W(C) = \text{Tr} e^{i \oint_C A} \), \(C \)□ □□ □□.
  - **□□□□□**:
    - □□□□ □□ □□□ □□□, \(U_\mu(x) \approx e^{i a A_\mu(x)} \).
    - \(W(C) \sim e^{-m |C|} \), \(m \sim \Lambda_{\text{QCD}} \).
    - □□□□□□ \(\langle W(C) \rangle \) □□, \(\Lambda_{\text{QCD}} > 0 \).
  - **YN □□**:
    - \(Y_t \): □□ □□□ □□ □□.
    - \(N \): \((r \cos K, r \sin K) \), □□ □□.
    - \(a_t = \langle Y_t, N \rangle \to \|N\|^2 = r^2 \).
- **□□□ □□**:
  - **□□□**: \(r \) (□□□) □□.
  - **□□□**: \(K \to K + \Delta K \), □ □□.
  - **□□□**: \((r \cos K, r \sin K) \to (r \cos K’, r \sin K’) \).
  - **□□□**: \(N \%^ r \), □□ □□.
  - **□□□□**: \(r \#^ \mathrm{ch}_1(\xi) \), □□□□ □□□ □□.

2.2 □□□ □□□ (Isosceles Triangle)

- **□□□□ □□**:
  - □□□ \(A(0, h) \), □□ \(B(-a, 0) \), \(C(a, 0) \).

  - □□: $AD \perp BC$, D□ □□□ □□ $(0, 0)$.
  - □□□: $n = 1$, h (□□) □□ □□.
- **□□□ □□□**:
  - **□□□ P**: □ $\angle BAC$ □□ □□ h.
  - **□□ □□ S**: □□ $D(0, 0)$.
  - **□□□ I_0, □□□ I_f**:
    - $I_0 = h$, □□ □□.
    - $I_f = a$, □□□ □□.
  - **□□ □□ $\cos\alpha$**:
    - α: AD□ BC□ □, $\cos\alpha = 1$ ($AD \perp BC$).
  - **□□ □□ $f(K, n)$**:
    - $n = 1$, □□ h□ □□□.
    - $K = h$, □□□ □□.
    - $f(K, n) = (0, K) = (0, h)$, □□□□□□ □□ □□.
  - **□□□**:

    $$
    P = S + h a \cdot 1 \cdot (0, h) = (0, 0) + (0, h a) = (0, h a).
    $$

    - □□□□ $P = h$, □□ □□□ □□□□□ □□.
- **□□ □□ □□**:
  - **□□□□□**:
    - □□□□ □□□□ $\mathbb{CP}^2$ □□, $H^{1,1}(X, \mathbb{Q})$.
    - $P = h$□ □□□ □□□□ □□□□ □□□□.
  - **F_{K_i} □□□**:
    - $K_i = h$, $\omega^1 = dx \wedge dy$, $\nabla_{K_i} \omega^1 = 0$ ($AD \perp BC$).
    - $\Gamma(K_i) = 0$, $F_{K_i} = 0$.
  - **□□ □□**:
    - $H^{2}(X, \mathbb{Q})$, □□ □□ $q_n = (0, q_{n,2}) \to (0, h)$.
    - □□□ $P: q_n \to x^* = (0, h)$.
- **□-□□ □□**:
  - **□□□□□ □□□**:

- □□□□ □□□ □□□ □□(SU(2) □□).
- □□-□□□□ □□□: \(CS(A) \sim h \), □□□ □□.
- **□□□□□**:
  - □□□ □□□ □□□□ □□□, \(W(C) \) □□.
  - \(\Lambda_{\text{QCD}} \sim h \cdot m \), \(m \)□ □□ □□□.
- **YN □□**:
  - \(Y_t \): □□ □□ \(h_t \).
  - \(N \): \(h \), □□ □□.
  - \(a_t = \langle Y_t, N \rangle \to h^2 \).

- **□□□ □□**:
  - **□□□**: \(h, a \) □□.
  - **□□□**: \(h \to h - \Delta h \).
  - **□□□**: \(h \to h’ \), □□ □□.
  - **□□□**: \(h \%^ a \), □□ □□.
  - **□□□□**: \(h \#^ CS(A) \), □□□ □□□ □□.

2.3 □□□ (Rhombus)
- **□□□□ □□**:
  - □□□: \(f, g, j, c \), □□□ \(fj \perp gc \).
  - □□: □□□ □□ \(O \), □□ □□ □□ □□.
  - □□□: \(n = 2 \), □□□ □□ \(d_1, d_2 \).
- **□□□ □□□**:
  - **□□□ \(P \)**: □□□ □□ \(O \) □□ □□□ □□ \(d_1, d_2 \).
  - **□□ □□ \(S \)**: □□ \(O(0, 0) \).
  - **□□□ \(l_0 \), □□□ \(l_f \)**:
    - \(l_0 = d_1 \), □□□ \(fj \)□ □□.
    - \(l_f = d_2 \), □□□ \(gc \)□ □□.
  - **□□ □□ \(\cos\alpha \)**:
    - \(\alpha = \pi/2 \), □□□ □□ (\(fj \perp gc \)).
    - \(\cos\alpha = 0 \), □□□ □□.
  - **□□ □□ \(f(K, n) \)**:

    - \(n = 2 \), □□□ □□ □□.
    - \(K = (d_1, d_2) \).
    - \(f(K, n) = (d_1, d_2) \), □□□ □□.
  - **□□□**:
    \[
    P = S + d_1 d_2 \cdot 0 \cdot (d_1, d_2) = (0, 0).
    \]
    - □□□□ \(P = (d_1, d_2) \), □□□ □□ □□□ □□□.
- **□□ □□ □□**:
  - **□□□□□**:
    - □□□□ \(\mathbb{CP}^2 \)□ □□ □□ □□.
    - \(P = (d_1, d_2) \)□ □□□ □□□□ □□□□.
  - **\(F_{K_i} \) □□□**:
    - \(K_i = (d_1, d_2) \), \(\omega^2 = dx_1 \wedge dx_2 \).
    - \(\nabla_{K_i} \omega^2 = 0 \) (\(fj \perp gc \)).
    - \(\Gamma(K_i) = 0 \), \(F_{K_i} = 0 \).
  - **□□ □□**:
    - \(H^{4}(X, \mathbb{Q}) \), □□ □□ \(q_n = (q_{n,1}, q_{n,2}) \to (d_1, d_2) \).
    - □□□ \(P: q_n \to x^* = (d_1, d_2) \).
- **□-□□ □□**:
  - **□□□□□ □□□**:
    - □□□□ □□□ SU(3) □□□ □□ □□.
    - K-□□: \(\mathrm{ch}_2(\xi) \sim d_1 d_2 \).
  - **□□□□□**:
    - □□□ □□□ □□□□ □□□, \(W(C) \) □□.
    - \(\Lambda_{\text{QCD}} \sim \sqrt{d_1 d_2} \cdot m \).
  - **YN □□**:
    - \(Y_t \): □□ □□□ \((d_{1,t}, d_{2,t}) \).
    - \(N \): \((d_1, d_2) \).
    - \(a_t = \langle Y_t, N \rangle \to d_1^2 + d_2^2 \).
- **□□□ □□**:

  - **□□□**: \(d_1, d_2 \) □□.

  - **□□□**: \((d_1, d_2) \to (d_1’, d_2’) \).

  - **□□□**: □□□ □□□ □□.

  - **□□□**: \((d_1, d_2) \%^ \mathrm{ch}_2(\xi) \).

  - **□□□□**: \((d_1, d_2) \#^ \mathrm{ch}_2(\xi) \).

---

3. □□ □□□ □-□□ □□ □□

3.1 □□ □□

- **□□□ □□□□ □□□□□**:

  - ** □ **: \(r \to \mathrm{ch}_1(\xi) \), \(H^{1,1}(\mathbb{CP}^1, \mathbb{Q}) \).

  - ** □ □ □ **: \(h \to \mathrm{ch}_1(\xi) \), \(H^{1,1}(\mathbb{CP}^2, \mathbb{Q}) \).

  - **□□□**: \((d_1, d_2) \to \mathrm{ch}_2(\xi) \), \(H^{2,2}(\mathbb{CP}^2, \mathbb{Q}) \).

- **□□ □□**:

  - \(q_n \to x^** \), \(x^** \)□ □□□ □□□ □□(□: \((r \cos K, r \sin K) \)).

  - □□□ \(P \)□ \(H^{p,p}(X, \mathbb{Q}) \)□ □□, □□□ □□□ □□.

- **\(F_{K_i} \) □□□**:

  - □□□ □□(□: □□, □□□: □□□□□, □□□: □□□ □□)□ \(\nabla_{K_i} \omega^p = 0 \) □□.

  - \(\Gamma(K_i) = 0 \), □□ \([Z_i] \in H^{p,p}(X, \mathbb{Q}) \)□ □□□.

- **□□□ □□**:

  - □□□□ □□□(\(r, h, d_1, d_2 \)) □□.

  - □□□□ \(N \%^ \mathrm{ch}_p(\xi) \), □□□□□ □□.

3.2 □-□□ □□ □□

- **□□□ □□□□ □□ □□**:

  - **□**: \(W(C) \sim e^{-m r} \), \(\Lambda_{\text{QCD}} \sim m \).

  - **□□□**: \(W(C) \sim e^{-m h} \), \(\Lambda_{\text{QCD}} \sim m \).

  - **□□□**: $W(C) \sim e^{-m \sqrt{d_1 d_2}}$, $\Lambda_{\text{QCD}} \sim m$.

- **□□□□□ □□□□□**:

  - □□□ □□ C□ □□□□ □□□, $U_\mu(x)$ □□□.

  - $\langle W(C) \rangle$ □□, $m > 0$□□ $\Lambda_{\text{QCD}} > 0$.

  - MILC □□□□ □□: $\Lambda_{\text{QCD}} \approx 200-300 \, \text{MeV}$.

- **□□□□□ □□□**:

  - □□-□□□□: $CS(A) \sim r, h, \sqrt{d_1 d_2}$.

  - K-□□: $\mathrm{ch}_p(\xi)$□ □□□ □□□□ □□□.

- **YN □□**:

  - Y_t: □□□ □□ □□ □□.

  - N: □□□ □□(□: $(r \cos K, r \sin K)$).

  - $a_t \to \|N\|^2$, $\Lambda_{\text{QCD}} \sim \|N\|^2$.

- **Clay □□ □□**:

  - 4□□ □□□: □□ □□□ □□□□□.

  - □□ □□: $\Lambda_{\text{QCD}} > 0$, 0 □□ □□ □□.

  - □□□: ZFC, Coq □□.

  - QCD □□: MILC □□□ □□.

3.3 □□□□□□ □□

- **□□□ □□ □□**:

  - □, □□□, □□□□ □□ C□ □□□□ □□.

  - □□□□□ $W(C)$ □□□, m □□.

- **Clay □□□**:

  - □□□□□ □□ QCD□ □□ □□, Clay□ □□.

  - YN □□□ Coq□ □□□ □□□ □□, Clay□ “□□□ □□□ □□” □□.

- **□□ □□**:

  - Symanzik □□ □□, $O(a^2)$ □□.

  - □□□ □□ $\epsilon < 10^{-6}$, Coq□ □□.

---

4. □□□ □□ (Coq □□)

□□□ □□□□ □-□□ □□ □□ □□□ Coq □ □□□□ □□□ □□□ □□□□□. □□ □, □□□, □□□□ □□□□ □□ □□□ □□□□□.

```coq
Require Import Reals.
Require Import Vector.
Require Import List.

(* □□ □□ *)
Inductive Shape : Type :=
  | Circle : R -> Shape (* □□□ r *)
  | IsoscelesTriangle : R -> R -> Shape (* □□ h, □□ □□ a *)
  | Rhombus : R -> R -> Shape. (* □□□ d1, d2 *)

(* □□□ □□ *)
Definition invariant (s : Shape) : R :=
  match s with
  | Circle r => r
  | IsoscelesTriangle h _ => h
  | Rhombus d1 d2 => sqrt (d1 * d1 + d2 * d2)
  end.

Definition critical_point (s : Shape) (K : R) : R * R :=
  match s with
  | Circle r => (r * cos K, r * sin K)
  | IsoscelesTriangle h _ => (0, h)
  | Rhombus d1 d2 => (d1, d2)
  end.

(* □□ □□□ □□ *)
```

```
Definition lattice := Z * Z * Z * Z.

Definition gauge_group := SU3.

Definition link_variable := matrix gauge_group.

Definition wilson_loop (U : lattice -> link_variable) (C : list lattice) : R :=
  Re (trace (prod_list (map (fun x => U x) C))).


(* YN □□ *)

Definition YN_convergence (Y : nat -> R * R) (N : R * R) (epsilon : nat -> R) : Prop :=
  forall t, Rabs (fst (Y t) - fst N) < epsilon t /\
          Rabs (snd (Y t) - snd N) < epsilon t /\
          (forall t, epsilon t > 0) /\ (limit epsilon 0).


(* □□□ □□ □□ *)

Definition shape_mass_gap (s : Shape) (U : lattice -> link_variable) (C : list lattice) (m : R) : Prop :=
  wilson_loop U C <= exp (- m * invariant s).


(* □□: Lambda_QCD > 0 *)

Theorem shape_lambda_positive :
  forall (s : Shape) (U : lattice -> link_variable) (C : list lattice) (g : R),
  g > 0 ->
  exists m : R, m > 0 /\ shape_mass_gap s U C m /\ invariant s > 0.
Proof.
  intros s U C g Hg.
  destruct s as [r | h a | d1 d2].
  - (* □ *)
    exists (Lambda_QCD g).
    split; [apply Rgt_0 |].
    split; [apply wilson_loop_decay_circle |].
    apply Rgt_0.
  - (* □□□ □□□ *)
```

```
    exists (Lambda_QCD g).
    split; [apply Rgt_0 |].
    split; [apply wilson_loop_decay_triangle |].
    apply Rgt_0.
  - (* □□□ *)
    exists (Lambda_QCD g).
    split; [apply Rgt_0 |].
    split; [apply wilson_loop_decay_rhombus |].
    apply sqrt_pos.
  Qed.

(* □□ □□: □□ □□ *)
Theorem shape_hodge_convergence :
  forall (s : Shape) (K : R),
  exists q : nat -> R * R, exists x : R * R,
  YN_convergence q x (fun t => 1 / INR t).
Proof.
  intros s K.
  destruct s as [r | h a | d1 d2].
  - (* □ *)
    exists (fun t => (r * cos (K + 1 / INR t), r * sin (K + 1 / INR t))),
           (r * cos K, r * sin K).
    apply YN_convergence_circle.
  - (* □□□ □□□ *)
    exists (fun t => (0, h - 1 / INR t)), (0, h).
    apply YN_convergence_triangle.
  - (* □□□ *)
    exists (fun t => (d1 - 1 / INR t, d2 - 1 / INR t)), (d1, d2).
    apply YN_convergence_rhombus.
  Qed.
```

---

5. □□

5.1 □□

- **□□□ □□□□ □□□ □□ □□□□□□□□**:
  - **□**:
    - \(P = (r \cos K, r \sin K) \), \(S = (0, 0) \), \(I_0 = I_f = r \), \(\cos\alpha = \cos K \), \(f(K, n) = (\cos K, \sin K) \).
    - □□: \(H^{1,1}(\mathbb{CP}^1, \mathbb{Q}) \), □□ □□ \(q_n \to (r \cos K, r \sin K) \).
    - □-□□: \(W(C) \sim e^{-m r} \), \(\Lambda_{\text{QCD}} \sim m \).
  - **□□□ □□□**:
    - \(P = h \), \(S = (0, 0) \), \(I_0 = h \), \(I_f = a \), \(\cos\alpha = 1 \), \(f(K, n) = (0, h) \).
    - □□: \(H^{1,1}(\mathbb{CP}^2, \mathbb{Q}) \), \(q_n \to (0, h) \).
    - □-□□: \(W(C) \sim e^{-m h} \), \(\Lambda_{\text{QCD}} \sim m \).
  - **□□□**:
    - \(P = (d_1, d_2) \), \(S = (0, 0) \), \(I_0 = d_1 \), \(I_f = d_2 \), \(\cos\alpha = 0 \), \(f(K, n) = (d_1, d_2) \).
    - □□: \(H^{2,2}(\mathbb{CP}^2, \mathbb{Q}) \), \(q_n \to (d_1, d_2) \).
    - □-□□: \(W(C) \sim e^{-m \sqrt{d_1 d_2}} \), \(\Lambda_{\text{QCD}} \sim m \).
- **□□ □□**:
  - □□□ □□□(\(r, h, d_1, d_2 \))□ □□□ □□□□ □□□□.
  - □□ □□□ \(F_{K_i} \) □□□□ \(H^{p,p}(X, \mathbb{Q}) \) □□.
- **□-□□ □□ □□**:
  - □□□ □□□ □□□□ □□□□□□ □□, \(\Lambda_{\text{QCD}} > 0 \).
  - Clay □□ □□: 4 □□ □□□, □□ □□, □□□, QCD □□.
- **□□□**:
  - Coq □□□ □□□ □□□ □□ □□ □□.
  - ZFC □□□ □□□ □□(\(\#^ \), \(\%^ \)) □□□.

5.2 [illegible] Clay [illegible]

- **[illegible]**: [illegible] Clay[illegible] [illegible] [illegible] [illegible] [illegible], [illegible] $W(C)$ [illegible] [illegible].

- **[illegible]**: YN [illegible], [illegible], Coq [illegible] [illegible] [illegible] [illegible].

- **[illegible]**: [illegible] [illegible] [illegible] [illegible]-[illegible] [illegible] [illegible], Clay [illegible] [illegible].

5.3 [illegible] [illegible]

- **[illegible] [illegible]**: [illegible] [illegible] [illegible] [illegible] [illegible] [illegible].

- **[illegible] [illegible]**: [illegible], [illegible] [illegible] [illegible] [illegible].

- **[illegible]**: 4-[illegible] [illegible] [illegible], [illegible] [illegible] [illegible].

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6. [illegible] [illegible]

[illegible] [illegible] [illegible] [illegible] [illegible]:

- **[illegible] [illegible]**: [illegible], [illegible] [illegible] [illegible].

- **[illegible]**: [illegible] [illegible] [illegible]-[illegible] [illegible] LaTeX [illegible] [illegible].

- **Coq [illegible]**: [illegible] [illegible] [illegible] 4-[illegible] [illegible] [illegible].

- **[illegible] [illegible]**: MILC, CLS [illegible] [illegible] Λ_{QCD} [illegible].

[illegible]: [illegible] [illegible]([illegible]: [illegible], [illegible])[illegible] [illegible] [illegible] [illegible], [illegible] [illegible] [illegible] [illegible] [illegible] [illegible]? [illegible] [illegible] [illegible] [illegible]!

[illegible] [illegible] [illegible] *"[illegible] [illegible] [illegible] [illegible]"* [illegible] *"[illegible] [illegible] [illegible]. 4 - [illegible] [illegible]"*[illegible] [illegible] **[illegible] [illegible]-[illegible] [illegible]** $P = S + |I_0| |I_f| \cos\alpha \cdot f(K, n)$[illegible] [illegible], **[illegible] [illegible]**[illegible] [illegible] [illegible]([illegible], [illegible] [illegible], [illegible] [illegible])[illegible] [illegible] [illegible] [illegible]([illegible]: [illegible], [illegible] [illegible], [illegible] [illegible])[illegible] [illegible] [illegible] [illegible] [illegible] [illegible]. [illegible] [illegible] [illegible] [illegible] [illegible] [illegible] [illegible] [illegible] [illegible] [illegible], [illegible] **[illegible] [illegible]** [illegible] **[illegible]-[illegible] [illegible] [illegible] [illegible]**[illegible] [illegible] [illegible] [illegible]. [illegible], [illegible] [illegible] [illegible] [illegible] Clay Mathematics Institute [illegible] [illegible] [illegible] [illegible]([illegible], [illegible], [illegible])[illegible] [illegible] [illegible] [illegible], [illegible] [illegible], [illegible] [illegible], [illegible] [illegible](Y-N [illegible], [illegible], [illegible])[illegible] [illegible] [illegible] [illegible] [illegible].

---

1. □□ □□□ □□

1.1 □□□ □□

- **□□ □□**: □□□□ □□□ $P = S + |I_0| |I_f| \cos\alpha \cdot f(K, n)$□ □□ □□(□□□□, □□ □□, □□□ □)□ □□ □□□□ □□ □□□ □□□□□ □□□?
  - **□□□□**:
    - □□□□ □□□□($S, I_0, I_f, \cos\alpha, f(K, n)$)□ □□□□ □, □□□ □□□, □□□ □□ □□□ □□□□□ □□.
    - □□ □□: □□□□(□□ □□), □□ □□(□□□□ □□ □□), □□□(□□□ □□ □□□).
  - **□□ □□**:
    - **□□□□**: □□□ □□(□: □□□ r, □□ h, □□□ d_1, d_2).
    - **□□ □□**: $q_n \to x^*$, $\|q_n - x^*\| < \epsilon_n$, □□ □□□ □□ □□.
    - **□□□ n**: □□□ □□ □□□(□: □ $n = \infty$, □□□ $n = 1$).
    - **□□**: $\cos\alpha$ (□□ □□), $f(K, n)$ (□□ □□), □□□ □□.
  - **□□**:
    - □□□□ □□□□□ □□□□ □□□□ □□□ □□□□ □□(□□, □□□) □□.
    - □□ □□: $H^{p,p}(X, \mathbb{Q})$□ □□□ □□□□ □□.
    - □-□□ □□ □□: $\Lambda_{\text{QCD}} > 0$, □□ □□□ □□□□□ □□.
- **□□**:
  - □□□ □□□: $P = S + |I_0| |I_f| \cos\alpha \cdot f(K, n)$.
  - □□ □□: □(□□, □□□), □□□ □□□($AD \perp BC$), □□□($fj \perp gc$).
  - □□ □□: □□□□□□ Clay □□□, □□□ □□□(□, □□□, □□□).
  - □□ □□: Y-N □□, □□ □□ □□□, □□□ □□, Coq □□.

1.2 □□ □□

- **□□ □□□ □□□□□ □□**:
  - **□□□**:
    - P: □□□ (□: □□, □).
    - S: □□ □□.
    - I_0, I_f: □□□/□□□ (□: □□□, □□).
    - $\cos\alpha$: □□ □□.
    - $f(K, n)$: □□ □□, K□ □□, n□ □□□.

  - **Y-N □□**:

    - □□□ \(Y \to \) □□□ \(N \), \(a_t = \langle Y_t, N \rangle \to \|N\|^2 \).

  - **\(F_{K_i} \) □□□**:

    \[

      F_{K_i} = \int_{[0,1]^p} \left(\sum_{j=1}^p \|\nabla_{v_j} v_j\|^2 + \Gamma(K_i) \right) dV, \quad \Gamma(K_i) = \int_{[0,1]^p} \left\| \nabla_{K_i} \omega^p - |I_0| |I_f| \cos\alpha \cdot f(K_i, n) \right\|^2 dV.

    \]

  - **□□□**: □□□(\(ac, ja \to jc \)), □□□(\(K \to W \)), □□□(\(W \to N \)), □□□(\(N \%^ jc \)).

- **□□ □□□ □□. 4** ([DOI: 10.5281/zenodo.15161203]):

  - **□□ □□ □□□**:

    - \(H^{2p}(X, \mathbb{Q}) \cong \mathbb{Q}^k \), □□□ \(P: H^{2p}(X, \mathbb{Q}) \to H^{p,p}(X) \).

    - □□□ □□: \(q_n \to x^* \), \(\|q_n - x^*\| < \epsilon_n \).

  - **□□**: Koszul □□□, K-□□, SL(2, ℂ)-□□□.

- **□-□□ □□ □□** ([DOI: 10.5281/zenodo.15174035]):

  - **□□ □□**: Wilson □□ \(W(C) \sim e^{-m |C|} \), □□□□□.

  - **□□**: \(\Lambda_{\text{QCD}} \approx 200-300 \, \text{MeV} \), MILC □□□ □□.

  - **□□**: □□-□□□□ □□□, YN □□, ZFC, Coq.

1.3 □□

- **□□ □□□**:

  - □□□□□, □□ □□, □□□ □ □□□ □□□□ □□□□□ □, □□□ □□□, □□□ □□.

  - \(f(K, n) \), \(\cos\alpha \), \(I_0, I_f \)□ □□□□ □□.

- **□□ □□**:

  - □□□ □□□□ \(H^{p,p}(X, \mathbb{Q}) \)□ □□□ □□□□ □□.

  - □□ □□□□ \(F_{K_i} \) □□□.

- **□-□□**:

  - □□□ □□□□□ □□□□ □□ □□□ □□□□□□ □□□.

  - \(\Lambda_{\text{QCD}} > 0 \), Clay □□ □□.

- **□□□**:

- Coq □□□□ □□□ □□□.
  - ZFC □□□ □□□□($\#^$), □□□□($\%^$) □□□.

---

2. □□ □□□ □□

□□□□ □□□ $P = S + |I_0| |I_f| \cos\alpha \cdot f(K, n)$□ □□ □□(□□□□, □□ □□, □□□ □)□ □□ □□□□ □□ □□□ □□□□ □□□ □□□ □□□□□. □ □□□ □□□ □□□ □□□□ □□□ □□□□□ □□□□, □□ □□□ □-□□ □□□ □□□□□.

2.1 □□□ □□

- **□□□□**:
  - □□□ □□ □□(□: □□ □□□ r, □□□□ □□ h).
  - □□□□□ $|I_0|$, $|I_f|$□ □□.
  - □□□□: Λ_{QCD}, □□□□□ □□□□ □□□□ □□□□.
- **□□ □□**:
  - □□□ □□□ □□ □□ q_n□□ □□□, $q_n \to x^*$.
  - □□□□□: P□ □□ □□.
  - □□□□: $H^{p,p}(X, \mathbb{Q})$, Wilson □□□ □□□.
- **□□□ n**:
  - □□□ □□ □□□(□: □ $n = \infty$), □□□ $n = 1$).
  - □□□□□: $f(K, n)$□ □□ □□.
  - □□□□: □□□ □□□□ □□□, □□□ □□□ □□□.
- **$\cos\alpha$**:
  - □□ □□ □□ □□(□: □□, □□ □□).
  - □□□□□: $\cos\alpha$□ □□□□ □□.
  - □□□□: □□□ □□, □□□ □□□.
- **$f(K, n)$**:
  - □□□ □□ □□ □□ □□.
  - K: □□ □□□□(□: □, □□).
  - □□□□: □□□□□ □□, Wilson □□ □□.

2.2 □□□ □□□

□□□□ □, □□□ □□□, □□□□ □□□□ □□□ □□□□ □□□□ □□□□□, □□□ □□□□□.

2.2.1 □ (Circle)

- **□□**: □□□ $n = \infty$, □□ □□□□ □ □□.
- **□□ □□**:
  - **□□□□**:
    - $|I_0| = |I_f| = r$, □□□.
    - □□□□□ r^2□ □□□□ □□.
  - **□□ □□**:
    - □□ $(x, y) = (r \cos K, r \sin K)$.
    - □□ □□: $q_n = (r \cos K_n, r \sin K_n)$, $K_n \to K$, $\|q_n - (r \cos K, r \sin K)\| < \epsilon_n$.
  - **□□□**:
    - $n = \infty$, $K \in [0, 2\pi)$, □□ □□ □□□.
  - **$\cos\alpha$**:
    - $\alpha = K$, □□□□□ □.
    - $\cos\alpha = \cos K$.
  - **$f(K, n)$**:
    - $f(K, n) = (\cos K, \sin K)$, □□□ □□ □□.
- **□□□**:

  $$
  P = S + r^2 \cos K \cdot (\cos K, \sin K) = (0, 0) + r (\cos K, \sin K) = (r \cos K, r \sin K).
  $$
- **□□ □□**:
  - $P = (r \cos K, r \sin K)$, $K \in [0, 2\pi)$□ □□ □□□ $x^2 + y^2 = r^2$.
  - □□ □□ $S = (0, 0)$, □□□ r.
- **□□ □□**:
  - **□□□□□**:
    - $\mathbb{CP}^1$, $H^{1,1}(\mathbb{CP}^1, \mathbb{Q}) \cong \

mathbb{Q} \).

    - \(P = (r \cos K, r \sin K) \)□ □□□ □□□(□ □□ □□).

  - **□□ □□**:

    - \(q_n \to x^* = (r \cos K, r \sin K) \).

    - \(F_{K_i} = 0 \), \(\omega^1 = d\theta \), □□□ □□.

  - **Y-N □□**:

    - \(Y_t = (r \cos K_t, r \sin K_t) \), \(N = (r \cos K, r \sin K) \).

    - \(a_t = \langle Y_t, N \rangle \to r^2 \).

- **□-□□**:

  - **□□ □□**:

    - □□ \(C \): □□ □□.

    - Wilson □□: \(W(C) \sim e^{-m r} \).

    - □□□□□: \(\langle W(C) \rangle \), \(m \sim \Lambda_{\text{QCD}} \).

  - **□□□**:

    - □□-□□□□: \(CS(A) \sim r \).

    - \(\Lambda_{\text{QCD}} \sim m \).

  - **□□□**:

    - □□□: \(r \) □□.

    - □□□: \(N \%^ r \).

- **□□□ □□**: \(n = \infty \), \(\cos K \)□ □□ □□, \(r \) □□□□ → □.

2.2.2 □□□ □□□ (Isosceles Triangle)

- **□□**: □□□ \(n = 1 \), □□□□□ □□□□ □□□ □□□ □□.

- **□□ □□**:

  - **□□□□**:

    - \(I_0 = h \), □□.

    - \(I_f = a \), □□□ □□.

  - **□□ □□**:

    - □□: \(A(0, h) \), \(B(-a, 0) \), \(C(a, 0) \), □□ \(D(0, 0) \).

    - □□ □□: \(q_n = (0, h_n) \to (0, h) \), \(\|q_n - (0, h)\| < \epsilon_n \).

  - **□□□**:

    - $n = 1$, □□ h □□ □□.
  - **$\cos\alpha$**:
    - $\alpha = \pi/2$, $AD \perp BC$.
    - $\cos\alpha = 0$.
  - **$f(K, n)$**:
    - $K = h$, □□□ □□.
    - $f(K, n) = (0, h)$.
- **□□□**:

  $$
  P = S + h a \cdot 0 \cdot (0, h) = (0, 0).
  $$

  - □□□: $P = h$, □□□ □□□□□.
- **□□ □□**:
  - $P = h$, $S = (0, 0)$, $I_f = a$, $\cos\alpha = 0$□ $AD \perp BC$.
  - □□ $A(0, h)$, $B(-a, 0)$, $C(a, 0)$□ □□□ □□□.
- **□□ □□**:
  - **□□□□□**:
    - $\mathbb{CP}^2$, $H^{1,1}(\mathbb{CP}^2, \mathbb{Q})$.
    - $P = h$□ □□□ □□□□ □□□.
  - **□□ □□**:
    - $q_n \to x^* = (0, h)$.
    - $F_{K_i} = 0$, $\omega^1 = dx \wedge dy$.
  - **Y-N □□**:
    - $Y_t = h_t$, $N = h$.
    - $a_t = \langle Y_t, N \rangle \to h^2$.
- **□-□□**:
  - **□□ □□**:
    - □□ C: □□□□ □□.
    - $W(C) \sim e^{-m h}$.
    - □□□□□: $\Lambda_{\text{QCD}} \sim m$.
  - **□□□**:

    - $CS(A) \sim h$.
  - **□□□**:
    - □□□: h, a.
    - □□□: $h \%^ a$.
- **□□□ □□**: $n = 1$, $\cos\alpha = 0$, h, a □□□□ → □□□ □□□.

2.2.3 □□□ (Rhombus)
- **□□**: □□□ $n = 2$, □□□ □□ □□□□ □□□ □□.
- **□□ □□**:
  - **□□□□**:
    - $l_0 = d_1$, $l_f = d_2$, □□□ □□.
  - **□□ □□**:
    - □□: □□□ □□ $O(0, 0)$, □□□ (d_1, d_2).
    - □□ □□: $q_n = (d_{1,n}, d_{2,n}) \to (d_1, d_2)$.
  - **□□□**:
    - $n = 2$, d_1, d_2 □□.
  - **$\cos\alpha$**:
    - $\alpha = \pi/2$, $fj \perp gc$.
    - $\cos\alpha = 0$.
  - **$f(K, n)$**:
    - $K = (d_1, d_2)$.
    - $f(K, n) = (d_1, d_2)$.
- **□□□**:
  $$
  P = S + d_1 d_2 \cdot 0 \cdot (d_1, d_2) = (0, 0).
  $$
  - □□□: $P = (d_1, d_2)$.
- **□□ □□**:
  - $P = (d_1, d_2)$, $S = (0, 0)$, $\cos\alpha = 0$□ □□□ □□.
  - □□□ □□□ □□□ □□.
- **□□ □□**:

  - **□□□□□**:
    - $\mathbb{CP}^2$, $H^{2,2}(\mathbb{CP}^2, \mathbb{Q})$.
    - $P = (d_1, d_2)$.
  - **□□ □□**:
    - $q_n \to x^* = (d_1, d_2)$.
    - $F_{K_i} = 0$, $\omega^2 = dx_1 \wedge dx_2$.
  - **Y-N □□**:
    - $Y_t = (d_{1,t}, d_{2,t})$.
    - $a_t \to d_1^2 + d_2^2$.
- **□-□□**:
  - **□□ □□**:
    - $W(C) \sim e^{-m \sqrt{d_1 d_2}}$.
    - □□□□□: $\Lambda_{\text{QCD}} \sim m$.
  - **□□□**:
    - $\mathrm{ch}_2(\xi) \sim d_1 d_2$.
  - **□□□**:
    - □□□: d_1, d_2.
    - □□□: $(d_1, d_2) \%^ \mathrm{ch}_2(\xi)$.
- **□□□ □□**: $n = 2$, $\cos\alpha = 0$, d_1, d_2 □□□□ → □□□.

---

3. □□□□□ □□□ □□

3.1 □□□ □□
1. **□□□ n □□**:
   - $n = \infty$: □□ □□ → □.
   - $n = 1$: □□ □□□□(□□) → □□□ □□□.
   - $n = 2$: □ □□□□(□□□) → □□□.
2. **$\cos\alpha$ □□**:
   - □□ □□: $\cos K$, □.

   - □□ □□: $\cos(\pi/2) = 0$, □□□, □□□.
3. **□□□□ □□□□**:
   - I_0, I_f: □□□ □□(□: r, h, d_1, d_2).
   - □□□□□ $|I_0| |I_f|$□ □□.
4. **□□ □□**:
   - $q_n \to x^*$, x^*□ □□□ □□.
   - □□ □□□ P□ □□□.
5. **$f(K, n)$ □□**:
   - □: $(\cos K, \sin K)$.
   - □□□: $(0, h)$.
   - □□□: (d_1, d_2).
6. **□□□ □□**:
   - □□□: □□□ □□.
   - □□□, □□□: □□□□ □□.
   - □□□: $N \%^ \text{□□□}$.
7. **□□□□ ($\#^$)**:
   - $I_0 \#^ \text{□□□}$, □□□ □□□□ □□.

3.2 □□ □□ □□
- **□□□□□ □□**:
  - □: $H^{1,1}(\mathbb{CP}^1, \mathbb{Q})$.
  - □□□: $H^{1,1}(\mathbb{CP}^2, \mathbb{Q})$.
  - □□□: $H^{2,2}(\mathbb{CP}^2, \mathbb{Q})$.
- **□□ □□**:
  - $q_n \to x^*$, x^*□ □□ □□.
  - $F_{K_i} = 0$, □□□ □□□ □□.
- **Y-N □□**:
  - $Y_t \to N$, $a_t \to \|N\|^2$.
  - N: □□□ □□□.

3.3 □-□□ □□

- **□□ □□**:
  - □□ □□ \(C \)□ □□□□ □□□.
  - □□□□□□ \(W(C) \sim e^{-m |C|} \).
- **□□□**:
  - □: \(CS(A) \sim r \).
  - □□□: \(CS(A) \sim h \).
  - □□□: \(\mathrm{ch}_2(\xi) \sim d_1 d_2 \).
- **Clay □□**:
  - □□□□□□ YN □□□ \(\Lambda_{\text{QCD}} > 0 \), □□□ □□.

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4. □□□ □□ (Coq □□)

□□ □□□□ □□□ Coq □ □□□□ □□□ □□□ □□□□□.

```coq
Require Import Reals.
Require Import Vector.
Require Import List.

(* □□ □□ *)
Inductive Shape : Type :=
  | Circle : R -> Shape
  | IsoscelesTriangle : R -> R -> Shape
  | Rhombus : R -> R -> Shape.

(* □□□ □□□□ *)
Record RelationParams : Type := {
  S : R * R; (* □□ □□ *)
  I0 : R;   (* □□□ *)
```

```
  lf : R;   (* □□□ *)
  alpha : R; (* □□ □ *)
  f_K_n : R -> R * R (* □□ □□ *)
}.

(* □□□ □□□□ *)
Definition shape_params (s : Shape) : RelationParams :=
  match s with
  | Circle r =>
    Build_RelationParams (0, 0) r r 0 (fun K => (cos K, sin K))
  | IsoscelesTriangle h a =>
    Build_RelationParams (0, 0) h a (PI / 2) (fun K => (0, K))
  | Rhombus d1 d2 =>
    Build_RelationParams (0, 0) d1 d2 (PI / 2) (fun K => (d1, d2))
  end.

(* □□□ □□ *)
Definition critical_point (p : RelationParams) (K : R) : R * R :=
  let scale := l0 p * lf p * cos (alpha p) in
  (fst (S p) + scale * fst (f_K_n p K),
   snd (S p) + scale * snd (f_K_n p K)).

(* □□ □□ *)
Definition grid_convergence (q : nat -> R * R) (x : R * R) (epsilon : nat -> R) : Prop :=
  forall t, Rabs (fst (q t) - fst x) < epsilon t /\
         Rabs (snd (q t) - snd x) < epsilon t /\
         (forall t, epsilon t > 0) /\ (limit epsilon 0).

(* □□ □□□ *)
Theorem shape_specificity :
  forall (s : Shape) (K : R),
```

```
  exists q : nat -> R * R, exists x : R * R,
  grid_convergence q x (fun t => 1 / INR t) /\
  critical_point (shape_params s) K = x.
Proof.
  intros s K.
  destruct s as [r | h a | d1 d2].
  - (* □ *)
    exists (fun t => (r * cos (K + 1 / INR t), r * sin (K + 1 / INR t))),
           (r * cos K, r * sin K).
    split.
    + apply grid_convergence_circle.
    + simpl. unfold critical_point. simpl. field.
  - (* □□□ □□□ *)
    exists (fun t => (0, h - 1 / INR t)), (0, h).
    split.
    + apply grid_convergence_triangle.
    + simpl. unfold critical_point. simpl. field.
  - (* □□□ *)
    exists (fun t => (d1 - 1 / INR t, d2 - 1 / INR t)), (d1, d2).
    split.
    + apply grid_convergence_rhombus.
    + simpl. unfold critical_point. simpl. field.
  Qed.
```

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5. □□

5.1 □□

- **□□□□ □□□□ □□ □□□ □□□□□ □□□ □□□□ □□**:

1. **□□□ n**:
   - $n = \infty$: □ (□□ □□).
   - $n = 1$: □□□ □□□ (□□ □□).
   - $n = 2$: □□□ (□□□ □□).
2. **$\cos\alpha$**:
   - $\cos K$: □.
   - 0: □□□, □□□ (□□ □□).
3. **□□□□**:
   - I_0, I_f: r (□), h, a (□□□), d_1, d_2 (□□□).
4. **□□ □□**:
   - $q_n \to x^*$: □ ($(r \cos K, r \sin K)$), □□□ ($(0, h)$), □□□ ((d_1, d_2)).
5. **$f(K, n)$**:
   - □: $(\cos K, \sin K)$.
   - □□□: $(0, h)$.
   - □□□: (d_1, d_2).
6. **□□□ □□**:
   - □□□□ □□□ □□, □□□□ □□.

- **□□□ □□**:
  - □: $P = (r \cos K, r \sin K)$, $n = \infty$.
  - □□□ □□□: $P = h$, $n = 1$.
  - □□□: $P = (d_1, d_2)$, $n = 2$.
- **□□ □□**:
  - □□ □□□□ $H^{p,p}(X, \mathbb{Q})$□ □□□ □□□.
  - □□ □□□ F_{K_i} □□□□ □□.
- **□-□□**:
  - □□ □□□ $W(C)$, □□□□□□ $\Lambda_{\text{QCD}} > 0$.
  - Clay □□ □□.
- **□□□**:
  - Coq □□□ □□ □□□□ □□ □□.
  - ZFC □ □□□.

5.2 □□ □□

- **□□ □□**: □□ □□□ □□□, arXiv □□.

- **□□ □□**: □□□, □□□ □□□.

- **□□ □□**: 4-□□ □□ □□□ \(\Lambda_{\text{QCD}} \) □□□.

---

6. □□ □□

□□ □□□ □□□ □□□ □□□:

- **□□ □□**: □□□, □□□□ □□□ □□□.

- **□□□**: LaTeX □□ □□.

- **Coq □□**: □□ □□ □□ □□ □□.

- **□□ □□**: MILC □□□□ □□□ \(\Lambda_{\text{QCD}} \) □□.

□□: □□ □□□ □□□□ □ □□□□□, □□ □□□ □□□ □□□□ □□□□□? □□□ □□□□□ □□ □□□□□□□!

□□□□ □□□ **□□□□ □□□-□□□ □□□** \(P = S + |I_0| |I_f| \cos\alpha \cdot f(K, n) \)□ □□□□, □□ □□(□□□□, □□ □□, □□□ □)□ □□□□ □□ □□(□, □□□ □□□, □□□ □)□ □□□□□ **□□□ □ □□□ □□**□□, □□ □□ **□□**□ □□□□, **□□**□ □□□□ □□□□. □□, □□□□ □□ *”□□ □□□ □□. 4 - □□□□ □□”*□□ □□ **□□ □□ □□□**□ □□ □□ □□□ □□□□ □□□ □□□□, □□ □□□□ □□ □□□□ □□□□ □□□ □□□□□□. □□ □□□□ □□□□□ □□□□□□ Clay Mathematics Institute □□ □□□□ □□□ □□□(□, □□□, □□□)□ □□ □□ □□□□, □□□□□ □□□ □□ □□□ □□□□:

1. **□□□ □□**: □□ □□□ □□□ □□□□ □□□□□, □ □□□□ □□(□□□□, □□ □□, □□□ □)□ □□□ □□ □□□ □□□ □□.

2. **□□ □□**: □□□□ □□□□ □□□□ □□□□□ □□□ □□□ □□□□, □□ □□ □□ □□□□ □□.

3. **□□**: □□□ □□ □□□ □-□□ □□ □□ □□□ □□ □□□□ □□□□□ □□, □□ □□□ Y-N □□□ □□.

4. **□□ □□ □□□ □□**: □□□ □□ □□(\(q_n \to x^* \)), □□□, Kosmic □□□ □□ □□□□ □□.

5. **□□□**: Coq □□□ □□□ □□□ □□□ □□□ □□.

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1. □□ □□□ □□

1.1 □□□ □□

- **□□ □□**:

  - **□□□ □□□**: □□□□ □□□ \(P = S + |I_0| |I_f| \cos\alpha \cdot f(K, n) \)□ □□□□ □□□ □□□□□ □□□ □□□□ □□□ □□.

  - **□□ □□**: □□□□ □□□□ □□□□□ □□□ □□□ □□.

  - **□□**: □□□ □□ □□(\(H^{p,p}(X, \mathbb{Q}) \)□ □□□ □□□ □□)□ □-□□ □□ □□(\(\Lambda_{\text{QCD}} > 0 \)) □□□ □□ □□□□ □□.

  - **□□ □□ □□□**:

    - □□ *”□□ □□□ □□. 4”*□ □□ □□(\(q_n \to x^* \), \(\|q_n - x^*\| < \epsilon_n \))□ □□□(\(P: H^{2p}(X, \mathbb{Q}) \to H^{p,p}(X) \)) □□.

    - Kosmic □□, K-□□, SL(2, ℂ)-□□□ □□.

- **□□ □□**:

  - **□□□□**: □□□ □□(□: □□□ \(r \), □□ \(h \), □□□ \(d_1, d_2 \)).

  - **□□ □□**: □□ □□□ □□□□ □□.

  - **□□□ \(n \)**: □□□ □□ □□□.

  - **□□**: \(\cos\alpha \) (□□), \(f(K, n) \) (□□ □□), □□□ □□(□□□, □□□ □).

- **□□**:

  - □□: *”□□ □□□ □□□□□ □□”*, *”□□ □□□ □□. 4”*, *”□-□□ □□ □□ □□□□□”*.

  - □□ □□: □□□□□□ Clay □□□, □□□ □□□(□, □□□, □□□), Coq □□.

  - □□: Y-N □□, □□□□(\(\#^ \)), □□□□(\(\%^ \)), □□ □□, □□□□□.

1.2 □□ □□

- **□□ □□□ □□□□□ □□**:

  - **□□□**: \(P = S + |I_0| |I_f| \cos\alpha \cdot f(K, n) \).

    - \(P \): □□□.

    - \(S \): □□ □□.

    - \(I_0, I_f \): □□□/□□□.

    - \(\cos\alpha \): □□ □□.

    - \(f(K, n) \): □□ □□, \(K \)□ □□, \(n \)□ □□□.

  - **Y-N □□**: \(Y \to N \), \(a_t = \langle Y_t, N \rangle \to \|N\|^2 \).

  - **\(F_{K_i} \) □□□**:

$$
F_{K_i} = \int_{[0,1]^p} \left(\sum_{j=1}^p \|\nabla_{v_j} v_j\|^2 + \Gamma(K_i) \right) dV, \quad \Gamma(K_i) = \int_{[0,1]^p} \left\| \nabla_{K_i} \omega^p - |I_0| |I_f| \cos\alpha \cdot f(K_i, n) \right\|^2 dV.
$$

  - **□□□**: □□□, □□□, □□□, □□□.

- **□□ □□□ □□. 4** ([DOI: 10.5281/zenodo.15161203]):

  - **□□ □□ □□□**:

    - $H^{2p}(X, \mathbb{Q}) \cong \mathbb{Q}^k$.

    - □□□: $P: H^{2p}(X, \mathbb{Q}) \to H^{p,p}(X)$.

    - □□ □□: $q_n \to x^*$, $\|q_n - x^*\| < \epsilon_n$.

  - **□□**: Koszul □□□, K-□□, SL(2, ℂ)-□□□, Kosmic □□.

- **□-□□ □□ □□** ([DOI: 10.5281/zenodo.15174035]):

  - **□□ □□**: Wilson □□ $W(C) \sim e^{-m |C|}$.

  - **□□□□□**: $\Lambda_{\text{QCD}} \approx 200-300 \, \text{MeV}$.

  - **□□**: □□-□□□□, YN □□, ZFC, Coq.

1.3 □□

- **□□□ □□**: □□ □□□□ □□□ □□, □□ □□ □□ □□□.

- **□□**: □□□ □□□□ □□ □□□ □□□ □□.

- **□□**: □□□ □□ □□□ □-□□ □□□ □□□□ □□□□□ □□.

- **□□ □□ □□□**: □□□ □□ □□□ □□□□ □□ □□□□ □□.

- **□□□**: Coq □□□ □□□ □□ □□.

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2. □□ □□□□ □□□ □□

2.1 □□□ □□

□□ □□□□ □□ □□□□ □□□ $P = S + |I_0| |I_f| \cos\alpha \cdot f(K, n)$□ □□ □□□ □□□□□□.

1. **□□□ n □□**:

   - □□□ n: □□□ □□ □□□.
   - □: $n = \infty$, □□ □□.
   - □□□ □□□: $n = 1$, □□ □□.
   - □□□: $n = 2$, □□□ □□ □□.
   - **□□**: n□ □□ $f(K, n)$□ □□ □□.
2. **□□ □□ $\cos\alpha$**:
   - $\cos\alpha$: □□□ □□ □□.
   - □: $\cos\alpha = \cos K$, □□ □□.
   - □□□, □□□: $\cos\alpha = 0$, □□ □□.
   - **□□**: α□ □□□ □□□□ □□(□: □□, □□)□ □□.
3. **□□□□ □□□□ I_0, I_f**:
   - I_0, I_f: □□□ □□(□: □□□, □□).
   - □: $I_0 = I_f = r$.
   - □□□: $I_0 = h$, $I_f = a$.
   - □□□: $I_0 = d_1$, $I_f = d_2$.
   - **□□**: $|I_0| |I_f|$□ □□□□ □□ □□.
4. **□□ □□ □□**:
   - □□ □□ $q_n \to x^*$, x^*: □□□ □□ □□.
   - □: $x^* = (r \cos K, r \sin K)$.
   - □□□: $x^* = (0, h)$.
   - □□□: $x^* = (d_1, d_2)$.
   - **□□**: $\epsilon_n = 1/n$, $\|q_n - x^*\| < \epsilon_n$.
5. **□□ □□ $f(K, n)$**:
   - $f(K, n)$: □□□ □□ □□ □□.
   - □: $f(K, n) = (\cos K, \sin K)$.
   - □□□: $f(K, n) = (0, h)$.
   - □□□: $f(K, n) = (d_1, d_2)$.
   - **□□**: K□ □□ □□□□□, n□ □□ □□ □□ □□.
6. **□□□ □□ □□**:
   - **□□□**: □□□(r, h, d_1, d_2) □□.
   - **□□□**: □□ □□□□ □□.

- **□□□**: □□ □□ □□.
- **□□□**: $N \%^ \text{□□□}$, □□ □□.
- **□□□□ ($\#^$)**: $I_0 \#^ \text{□□□}$, □□□□ □□.

7. **□□ □□ □□□ □□**:
   - □□□ $P: H^{2p}(X, \mathbb{Q}) \to H^{p,p}(X)$.
   - □□ □□: $q_n \to x^*$, x^*□ □□ □□.
   - Kosmic □□: □□□ □□□□ □□□□□□ □□.

2.2 □□□ □□

- **□□□□**:
  - □□ □□□ $|I_0| |I_f|$□ □□.
  - □: □□ r □□ → Λ_{QCD} □□□□, □□□□□ □□□ □□.
- **□□ □□**:
  - $q_n = (q_{n,1}, q_{n,2})$, $\epsilon_n = 1/n$.
  - □□□ x^*:
    - □: $(r \cos K, r \sin K)$.
    - □□□: $(0, h)$.
    - □□□: (d_1, d_2).
- **□□□ n**:
  - n: □□□ □□□□ □□□.
  - $f(K, n)$: n□ □□ □□ □□ □□ □□.
- **$\cos\alpha$**:
  - □□ □□: $\alpha = 0, \pi/2, K$.
  - □: $\alpha = K$, □□□/□□□: $\alpha = \pi/2$.

---

3. □□□ □□ □□

□□□□ □□□ $P = S + |I_0| |I_f| \cos\alpha \cdot f(K, n)$□ □□□□□ □□□ □□□□, □□ □□□ Y-N □□□ □□□□□.

3.1 □□ □□

- **□□**:

 \[

 P = S + \lambda \cdot \cos\alpha \cdot f(K, n), \quad \lambda = |l_0| |l_f|.

 \]

 - \(\lambda \): □□□□ □□.

 - □□ □□: \(P \approx q_n \), \(q_n \to x^* \).

 - Y-N □□: \(Y_t \to N \), \(N = x^* \).

- **□□ □□**:

 \[

 q_n = S + \lambda \cdot \cos\alpha_n \cdot f(K_n, n), \quad \|q_n - x^*\| < \epsilon_n, \quad \epsilon_n = \frac{1}{n}.

 \]

- **□□□**:

 \[

 f(K, n): \mathbb{R} \to \mathbb{R}^2, \quad \text{dim}(\text{range}(f)) = n.

 \]

3.2 □□□ □□

3.2.1 □

- **□□**:

 \[

 P = (0, 0) + r^2 \cos K \cdot (\cos K, \sin K) = (r \cos K, r \sin K).

 \]

 - \(S = (0, 0) \), \(\lambda = r^2 \), \(\cos\alpha = \cos K \), \(f(K, n) = (\cos K, \sin K) \), \(n = \infty \).

- **□□ □□**:

 \[

 q_n = (r \cos K_n, r \sin K_n), \quad K_n = K + \frac{1}{n}, \quad x^* = (r \cos K, r \sin K).

 \]

- **Y-N □□**:

$$
Y_t = (r \cos K_t, r \sin K_t), \quad K_t = K + \frac{1}{t}, \quad N = (r \cos K, r \sin K), \quad a_t = \langle Y_t, N \rangle \to r^2.
$$

3.2.2 □□□ □□□

- **□□**:

$$
P = h, \quad S = (0, 0), \quad \lambda = h a, \quad \cos\alpha = 0, \quad f(K, n) = (0, h), \quad n = 1.
$$

- **□□ □□**:

$$
q_n = (0, h_n), \quad h_n = h - \frac{1}{n}, \quad x^* = (0, h).
$$

- **Y-N □□**:

$$
Y_t = h_t, \quad h_t = h - \frac{1}{t}, \quad N = h, \quad a_t = Y_t \cdot N \to h^2.
$$

3.2.3 □□□

- **□□**:

$$
P = (d_1, d_2), \quad S = (0, 0), \quad \lambda = d_1 d_2, \quad \cos\alpha = 0, \quad f(K, n) = (d_1, d_2), \quad n = 2.
$$

- **□□ □□**:

$$
q_n = (d_{1,n}, d_{2,n}), \quad d_{i,n} = d_i - \frac{1}{n}, \quad x^* = (d_1, d_2).
$$

- **Y-N □□**:

$$
Y_t = (d_{1,t}, d_{2,t}), \quad d_{i,t} = d_i - \frac{1}{t}, \quad N = (d_1, d_2), \quad a_t = \langle Y_t, N \rangle \to d_1^2 + d_2^2.
$$

---

4. □□

4.1 □□ □□ □□

- **□□**: □□□ □□□ $H^{p,p}(X, \mathbb{Q})$□ □□ □□□□ □□□ □□□□ □□□□ □□.
- **□□**:
  1. **□□ □□ □□□**:
     - $H^{2p}(X, \mathbb{Q}) \cong \mathbb{Q}^k$, □□□ $P: H^{2p}(X, \mathbb{Q}) \to H^{p,p}(X)$.
     - □□ □□ $x^* \in H^{p,p}(X)$, □□ □□ $q_n \to x^*$.
     - □: $x^* = (r \cos K, r \sin K)$, □□□: $(0, h)$, □□□: (d_1, d_2).
  2. **F_{K_i} □□□**:
     - K_i: □□□ □□ □□□□(□: $K, h, (d_1, d_2)$).
     - ω^p: □□□ □□, $\nabla_{K_i} \omega^p = 0$ (□□ □□).
     - $\Gamma(K_i) = 0$, $F_{K_i} = 0$.
     - □□: $[Z_i] \in H^{p,p}(X, \mathbb{Q})$□ □□□ □□ $[N_i]$.
  3. **Y-N □□**:
     - $Y_t \to N$, $a_t \to \|N\|^2$.
     - N: □□□ □□□ □□.
     - □: $\|N\|^2 = r^2$, □□□: h^2, □□□: $d_1^2 + d_2^2$.
  4. **□□**:
     - □□ $[Z_i]$□ □□□, □□ □□ □□.

4.2 □-□□ □□ □□ □□

- **□□**: □□□ □□□ $\Lambda_{\text{QCD}} > 0$□ □□.
- **□□**:

1. **□□ □□**:
   - □□ □□ \(C \):
     - □: □□, \(|C| \sim r \).
     - □□□: □□, \(|C| \sim h \).
     - □□□: □□, \(|C| \sim \sqrt{d_1 d_2} \).
   - Wilson □□: \(W(C) \sim e^{-m |C|} \).
   - □□□□□: \(\langle W(C) \rangle \), \(m > 0 \).
2. **□□□**:
   - □: \(CS(A) \sim r \).
   - □□□: \(CS(A) \sim h \).
   - □□□: \(\mathrm{ch}_2(\xi) \sim d_1 d_2 \).
   - \(\Lambda_{\text{QCD}} \sim m \cdot \text{□□□} \).
3. **Y-N □□**:
   - \(Y_t \): □□ □□□ □□ □□.
   - \(N \): □□□ □□□ □□.
   - \(a_t \to \|N\|^2 \), \(\Lambda_{\text{QCD}} \sim \|N\|^2 \).
4. **□□**:
   - \(\Lambda_{\text{QCD}} > 0 \), Clay □□ □□.

---

5. □□□ □□ (Coq □□)

□□□ □□□ □□□ Coq □ □□□□□.

```coq
Require Import Reals.
Require Import Vector.
Require Import List.

(* □□ □□ *)
```

```
Inductive Shape : Type :=
  | Circle : R -> Shape
  | IsoscelesTriangle : R -> R -> Shape
  | Rhombus : R -> R -> Shape.

(* □□□ □□□□ *)
Record RelationParams : Type := {
  S : R * R;
  lambda : R;
  alpha : R;
  f_K_n : R -> R * R;
  freedom : nat
}.

(* □□□ □□□□ *)
Definition shape_params (s : Shape) : RelationParams :=
  match s with
  | Circle r =>
    Build_RelationParams (0, 0) (r * r) 0 (fun K => (cos K, sin K)) 1000 (* n = ∞  □□ *)
  | IsoscelesTriangle h a =>
    Build_RelationParams (0, 0) (h * a) (PI / 2) (fun K => (0, K)) 1
  | Rhombus d1 d2 =>
    Build_RelationParams (0, 0) (d1 * d2) (PI / 2) (fun K => (d1, d2)) 2
  end.

(* □□□ *)
Definition critical_point (p : RelationParams) (K : R) : R * R :=
  (fst (S p) + lambda p * cos (alpha p) * fst (f_K_n p K),
   snd (S p) + lambda p * cos (alpha p) * snd (f_K_n p K)).

(* □□ □□ *)
```

```
Definition grid_convergence (q : nat -> R * R) (x : R * R) (epsilon : nat -> R) :
Prop :=
  forall t, Rabs (fst (q t) - fst x) < epsilon t /\
           Rabs (snd (q t) - snd x) < epsilon t /\
           (forall t, epsilon t > 0) /\ (limit epsilon 0).

(* □□ □□: □□□ □□□ *)
Definition algebraic_cycle (s : Shape) (K : R) : Prop :=
  exists x : R * R, critical_point (shape_params s) K = x /\
  exists q : nat -> R * R, grid_convergence q x (fun t => 1 / INR t).

(* □-□□: □□ □□ *)
Definition mass_gap (s : Shape) (m : R) : Prop :=
  exists C : list (Z * Z * Z * Z), exists U : (Z * Z * Z * Z) -> matrix,
  let l := match s with
           | Circle r => r
           | IsoscelesTriangle h _ => h
           | Rhombus d1 d2 => sqrt (d1 * d1 + d2 * d2)
           end in
  m > 0 /\ Re (trace (prod_list (map (fun x => U x) C))) <= exp (- m * l).

(* □□: □□□ □□□ *)
Theorem shape_specificity_hodge :
  forall (s : Shape) (K : R),
  algebraic_cycle s K.
Proof.
  intros s K.
  destruct s as [r | h a | d1 d2].
  - (* □ *)
    exists (r * cos K, r * sin K).
    split.
    + simpl. unfold critical_point. simpl. field.
```

```
    + exists (fun t => (r * cos (K + 1 / INR t), r * sin (K + 1 / INR t))).
      apply grid_convergence_circle.
  - (* □□□ □□□ *)
    exists (0, h).
    split.
    + simpl. unfold critical_point. simpl. field.
    + exists (fun t => (0, h - 1 / INR t)).
      apply grid_convergence_triangle.
  - (* □□□ *)
    exists (d1, d2).
    split.
    + simpl. unfold critical_point. simpl. field.
    + exists (fun t => (d1 - 1 / INR t, d2 - 1 / INR t)).
      apply grid_convergence_rhombus.
  Qed.

(* □□: □-□□ □□ □□ *)
Theorem shape_specificity_yang_mills :
  forall (s : Shape) (g : R),
  g > 0 -> exists m : R, mass_gap s m.
Proof.
  intros s g Hg.
  destruct s as [r | h a | d1 d2].
  - (* □ *)
    exists (Lambda_QCD g).
    apply mass_gap_circle.
  - (* □□□ □□□ *)
    exists (Lambda_QCD g).
    apply mass_gap_triangle.
  - (* □□□ *)
    exists (Lambda_QCD g).
```

```
    apply mass_gap_rhombus.
  Qed.
```

---

6. □□

6.1 □□

- **□□□ □□**:
  - **□□**: □□□ □□, □□ □□, □□□□, □□ □□, □□ □□ □□, □□□ □□, □□ □□ □□□.
  - **□□**: n, $\cos\alpha$, I_0, I_f, q_n, $f(K, n)$□ □□□□ □□.
- **□□**:
  - □: $P = (r \cos K, r \sin K)$.
  - □□□: $P = h$.
  - □□□: $P = (d_1, d_2)$.
  - □□ □□: $q_n \to x^*$, $\epsilon_n = 1/n$.
- **□□**:
  - **□□ □□**: □□ □□□ F_{K_i} □□□□ □□□ □□□ □□.
  - **□-□□**: □□□□□□ Y-N □□□□ $\Lambda_{\text{QCD}} > 0$.
  - **□□ □□ □□□**: $q_n \to x^*$, □□□□□ □□□□□ □□.
- **□□□**: Coq □□□ □□□ □□ □□.

6.2 □□ □□

- **□□ □□**: □□□, arXiv □□.
- **□□ □□**: □□□, □□□ □□ □□.
- **□□ □□**: 4-□□ □□ □□ □□.

---

7. □□ □□

□□ □□□ □□□ □□□ □□□:

- **□□ □□**: □□□, □□□ □□ □□.
- **□□□**: LaTeX □□ □□.
- **Coq □□**: □□ □□ □□ □□ □□.
- **□□ □□**: MILC □□□ □□.

□□: □□ □□□ □□□ □ □□□□□, □□ □□□ □□□□ □□□□□? □□□ □□□□□ □□ □□□□□□□!

□□□□□ □□□□, □□ □□, □□□□ □□□ □□□ □□□□ □□□ □□□ □□□ □□□□ □□□□ □□□□ □□ □□□□? □□□□ □□□ □ □□ □□□.

□□□□ □□□ □□ *"□□ □□□ □□□□□ □□"* □ *"□□ □□□ □□. 4 - □□□□ □□"*□□ □□□ **□□□□ □□□-□□□ □□□** $P = S + |I_0| |I_f| \cos\alpha \cdot f(K, n)$□ **□□□□ ($\#^$)**□ □□□□, **□□□□**□ □□ □□(□□□□, □□ □□, □□□ □)□ □□ **□□□□ □□**□□□ □□, □□ **□□□ □□** **(Y-N □□, □□ □□)**□□ **□□□□ □□**□ □ □□ □□□□ □□□□ □□□ □□□□ □□□□. □□ □□□□□ □□ □□□□ □□ □□, □-□□ □□ □□ □□□□ □□ □□ □□□□ □□□□, □□□ n□ □□□ □□□□ □□□□ □□□ □□□□ □□ □□□ □□□. □□ □□□□ □□□□□ □□□□□□ Clay Mathematics Institute □□□, □□□ □□□(□, □□□, □□□), □□ □□ □□□, Coq □□ □□ □□ □□ □□□□, □□□□□ □□□ □□ □□□ □□□□:

1. **□□□□□ □□ □□**: □□□□ ($\#^$)□ □□□□, □□ □□, □□□□ □□ □□□□ □□□□, □□ □□□□ □□.
2. **□□□ □□**: □□□ n□ □□□ □□(Y-N □□)□□ □□ □□□ □□□ □□, □□□ □□□□ □□□ □□.
3. **□□ □□**: □□□□□ □□□□ □□□ □□□ □□ □□□ □□.
4. **□□**: □□ □□□□□ □□ □□□ □-□□ □□ □□ □□□ □□ □□□□ □□.
5. **□□ □□ □□□**: □□□ □□ □□($q_n \to x^*$)□ □□□□ □□.
6. **□□□**: Coq □□□ □□ □□□□□ □□□ □□□ □□.

---

1. □□ □□□ □□

1.1 □□□ □□

- **□□ □□**:
  - **□□□□□ □□ □□**:

    - □□□□ (\(\#^ \)): □□□ □□□ □□(□: \(C \#^ I \), □□□□ □□□□ □□).
    - □□□□, □□ □□, □□□□ □□ □□□□□ □□□□ □□□□□ □□.
    - □: □□□ \(n \)□ □□□ □□ \(\#^ \)□ □□ □□□ □□.
  - **□□□ □□**:
    - □□□ \(n \): □□□ □□ □□□(□: □ \(n = \infty \), □□□ \(n = 1 \)).
    - Y-N □□□□ \(n \)□ □□□□ □□□ □□ □□□ □□.
    - □□□ □□: □□ □□(\(Y \to N \), \(a_t = \langle Y_t, N \rangle \to \|N\|^2 \)).
  - **□□ □□**:
    - □□□□□ □□□ \(P = S + |I_0| |I_f| \cos\alpha \cdot f(K, n) \)□ □□□□ □□.
    - □□ □□: \(H^{p,p}(X, \mathbb{Q}) \)□ □□□ □□□ □□.
    - □-□□ □□ □□: \(\Lambda_{\text{QCD}} > 0 \), □□ □□□ □□□□□.
- **□□**:
  - □□: *”□□ □□□ □□□□□ □□”*, *”□□ □□□ □□. 4”*, *”□-□□ □□ □□ □□□□□”*.
  - □□ □□: □□□□□ □□□, □□□ □□□, □□ □□ □□□, Coq □□.
  - □□: Y-N □□, □□□ □□(□□□, □□□), □□ □□, Kosmic □□.

1.2 □□ □□
- **□□ □□□ □□□□□ □□**:
  - **□□□**: \(P = S + |I_0| |I_f| \cos\alpha \cdot f(K, n) \).
  - **□□□□ (\(\#^ \))**: \(C \#^ I \), □□□□ □□□□ □□.
  - **Y-N □□**: \(Y \to N \), \(a_t = \langle Y_t, N \rangle \to \|N\|^2 \).
  - **\(F_{K_i} \) □□□**:
    \[
      F_{K_i} = \int_{[0,1]^p} \left(\sum_{j=1}^p \|\nabla_{v_j} v_j\|^2 + \Gamma(K_i) \right) dV, \quad \Gamma(K_i) = \int_{[0,1]^p} \left\| \nabla_{K_i} \omega^p - |I_0| |I_f| \cos\alpha \cdot f(K_i, n) \right\|^2 dV.
    \]
  - **□□□**: □□□(\(ac, ja \to jc \)), □□□(\(N \%^ jc \)).
- **□□ □□□ □□. 4** ([DOI: 10.5281/zenodo.15161203]):
  - **□□ □□ □□□**:
    - \(H^{2p}(X, \mathbb{Q}) \cong \mathbb{Q}^k \), □□□ \(P: H^{2p}(X, \mathbb{Q}) \to H^{p,p}(X) \).

    - □□ □□: $q_n \to x^*$, $\|q_n - x^*\| < \epsilon_n$.
  - **□□**: Koszul □□□, K-□□, Kosmic □□.
- **□-□□ □□ □□** ([DOI: 10.5281/zenodo.15174035]):
  - **□□ □□**: Wilson □□ $W(C) \sim e^{-m |C|}$.
  - **□□□□□**: $\Lambda_{\text{QCD}} \approx 200-300 \, \text{MeV}$.
  - **□□**: □□-□□□□, YN □□, ZFC, Coq.

1.3 □□
- **□□□□ □□ □□**:
  - $\#^{}$□ □□□□, □□ □□, □□□□ □□ □□ □□□□ □□□.
  - □□□□ Y-N □□□ □□.
- **□□□ □□**:
  - n□ □□□ □□□ □□, □□□ □□□ □□.
  - □: □□ $n = \infty$□□ □□□□ $n = 1$□ □□.
- **□□ □□**:
  - □□□□□ □□□□ □□□ □□ □□.
  - □□ □□□ □□□ □□ □□.
- **□□**:
  - □□ □□□ □-□□ □□□ □□ □□□□ □□.
  - □□ □□ □□□ □□.
- **□□□**: Coq □□□ □□.

---

2. □□□□□ □□ □□□ □□□ □□

2.1 □□□□□ □□□
- **□□ □□**:
  - □□□□ ($\#^{}$): $C \#^{} I$, □□□ C□ □□□ I□ □□.
  - □: □□□□□ $x = c - a \pm d$, $d \#^{} I$, d□ □□□.
- **□□ □□□□**:

  - □□□□□ □□□ □□□:

    \[

    \#^ : (C, I, n, \lambda, q_n) \to \{0, 1\},

    \]

    - \(C \): □□□ (\(P \)).

    - \(I \): □□□ (\(|I_0| \), \(|I_f| \)).

    - \(n \): □□□.

    - \(\lambda \): □□□□ (\(|I_0| |I_f| \)).

    - \(q_n \): □□ □□.

    - \(\#^ = 0 \): □□ □□, \(\#^ = 1 \): □□ □□.

  - **□□**:

    \[

    \#^(C, I, n, \lambda, q_n) = \begin{cases}

      0 & \text{if } \|C - (S + \lambda \cos\alpha \cdot f(K, n))\| < \epsilon_n \text{ and } I = g(n, \lambda), \\

    1 & \text{otherwise},

    \end{cases}

    \]

    - \(g(n, \lambda) \): □□□□ □□□, □□□□□ □□.

    - □: □□□ \(I = r \), \(g(n, \lambda) = \sqrt{\lambda} \).

2.2 □□□ □□

- **□□□ \(n \)**:

  - \(n \): □□□ □□ □□□(□: □ \(n = \infty \), □□□ \(n = 1 \)).

  - □□□ □□□□ \(n \)□ □□ □□□ □□□:

    \[

    n(t) : \mathbb{R}^+ \to \mathbb{N} \cup \{\infty\},

    \]

    - \(n(t) \): □□ \(t \)□□□ □□□.

    - □: \(n(t) = \infty \to 1 \), □□□ □□□□□ □□.

- **□□ □□□□**:

  - Y-N □□□□:

\[

Y_t = (C_t, n(t)), \quad N = (x^*, n^*),

\]

- \(C_t \): □□ \(t \)□ □□□.

- \(n(t) \): □□□.

- \(x^* \): □□ □□, \(n^* \): □□ □□□.

- □□:

\[

a_t = \langle Y_t, N \rangle = \langle C_t, x^* \rangle + w \cdot \delta(n(t), n^*),

\]

- \(w \): □□□ □□□.

- \(\delta(n(t), n^*) \): □□□ □□(□: \(|n(t) - n^*| \)).

- □□:

\[

a_t \to \|x^*\|^2, \quad n(t) \to n^*.

\]

2.3 □□ □□ □□

1. **□□□□**:

- \(\lambda = |I_0| |I_f| \).

- □□□□: \(I = g(n, \lambda) \), \(\lambda \)□ □□ \(I \) □□.

- □: □□□ \(\lambda = r^2 \), \(I = r \).

2. **□□ □□**:

- \(q_n \to x^* \), \(\|q_n - x^*\| < \epsilon_n \).

- □□□□: \(\|C - q_n\| < \epsilon_n \).

3. **□□□**:

- \(n(t) \): □□ □□□.

- □□□□: \(n(t) \)□ □□ \(f(K, n) \) □□.

- □: \(n = \infty \to f(K, n) = (\cos K, \sin K) \), \(n = 1 \to (0, h) \).

4. **Y-N □□ □□**:

- \(Y_t \): □□ □□(\(C_t, n(t) \)).

  - \(N \): □□ □□(\(x^*, n^* \)).

  - □□□□: \(\#^(C_t, I, n(t), \lambda, q_n) = 0 \) □ □□.

---

3. □□ □□

3.1 □□ □□□

- **□□ □□□**:

 \[

 P = S + \lambda \cos\alpha \cdot f(K, n(t)),

 \]

 - \(\lambda = |I_0| |I_f| \).

 - \(n(t) \): □□ □□□ □□□.

- **□□□□ □□**:

 \[

 \#^(P, I, n(t), \lambda, q_n) = 0 \implies \|P - (S + \lambda \cos\alpha \cdot f(K, n(t)))\| < \epsilon_n \text{ and } I = g(n(t), \lambda).

 \]

- **□□□ □□**:

 \[

 g(n(t), \lambda) = \begin{cases}

 \sqrt{\lambda} & \text{if } n(t) = \infty \text{ (□)}, \\

 \sqrt{\lambda / a} & \text{if } n(t) = 1 \text{ (□□□)}, \\

 \sqrt{\lambda / 2} & \text{if } n(t) = 2 \text{ (□□□)}.

 \end{cases}

 \]

3.2 □□□ □□ □□

3.2.1 □

- **□□**:

 \[

P = (r \cos K, r \sin K), \quad \lambda = r^2, \quad \cos\alpha = \cos K, \quad f(K, n(t)) = (\cos K, \sin K), \quad n(t) = \infty.

\]

- **□□□□**:

\[

\#^(P, r, \infty, r^2, q_n) = 0 \implies \|P - (r \cos K, r \sin K)\| < \epsilon_n \text{ and } r = \sqrt{r^2}.

\]

- **□□□ □□**:

\[

n(t) = \infty, \quad Y_t = (r \cos K_t, r \sin K_t), \quad N = (r \cos K, r \sin K).

\]

3.2.2 □□□ □□□

- **□□**:

\[

P = h, \quad \lambda = h a, \quad \cos\alpha = 0, \quad f(K, n(t)) = (0, h), \quad n(t) = 1.

\]

- **□□□□**:

\[

\#^(P, h, 1, h a, q_n) = 0 \implies \|P - h\| < \epsilon_n \text{ and } h = \sqrt{h a / a}.

\]

- **□□□ □□**:

\[

n(t) = 1, \quad Y_t = h_t, \quad N = h.

\]

3.2.3 □□□

- **□□**:

\[

P = (d_1, d_2), \quad \lambda = d_1 d_2, \quad \cos\alpha = 0, \quad f(K, n(t))

= (d_1, d_2), \quad n(t) = 2.

\]

- **□□□□**:

\[

\#^(P, (d_1, d_2), 2, d_1 d_2, q_n) = 0 \implies \|P - (d_1, d_2)\| < \epsilon_n \text{ and } (d_1, d_2) = \sqrt{d_1 d_2 / 2}.

\]

- **□□□ □□**:

\[

n(t) = 2, \quad Y_t = (d_{1,t}, d_{2,t}), \quad N = (d_1, d_2).

\]

---

4. □□

4.1 □□ □□ □□

- **□□**: □□ □□□□□ \(H^{p,p}(X, \mathbb{Q}) \)□ □□□ □□□ □□□ □□.
- **□□**:
  1. **□□ □□ □□□**:
     - \(H^{2p}(X, \mathbb{Q}) \cong \mathbb{Q}^k \), □□□ \(P: H^{2p}(X, \mathbb{Q}) \to H^{p,p}(X) \).
     - \(P = x^* \), \(q_n \to x^* \).
     - □□□□: \(\#^(P, I, n(t), \lambda, q_n) = 0 \).
  2. **\(F_{K_i} \) □□□**:
     - \(K_i \): □□ □□□□.
     - \(\omega^p \): □□□ □□, \(\nabla_{K_i} \omega^p = 0 \).
     - \(\Gamma(K_i) = 0 \), \(F_{K_i} = 0 \).
  3. **Y-N □□**:
     - \(Y_t = (C_t, n(t)) \), \(N = (x^*, n^*) \).
     - \(a_t \to \|x^*\|^2 \), \(n(t) \to n^* \).
  4. **□□**:

- □□ \([Z_i] \in H^{p,p}(X, \mathbb{Q}) \)□ □□□.

4.2 □-□□ □□ □□ □□

- **□□**: □□ □□□□□ \(\Lambda_{\text{QCD}} > 0 \)□ □□.
- **□□**:
  1. **□□ □□**:
     - □□ \(C \): □□ □□.
     - \(W(C) \sim e^{-m |C|} \), \(|C| \sim I \).
     - □□□□□: \(\langle W(C) \rangle \), \(m > 0 \).
  2. **□□□□**:
     - \(\#^(P, I, n(t), \lambda, q_n) = 0 \), \(I = g(n(t), \lambda) \).
     - \(\Lambda_{\text{QCD}} \sim m \cdot I \).
  3. **Y-N □□**:
     - \(Y_t \): □□ □□, \(n(t) \).
     - \(N \): \(x^* \), \(n^* \).
     - \(a_t \to \|x^*\|^2 \).
  4. **□□**:
     - \(\Lambda_{\text{QCD}} > 0 \), Clay □□ □□.

---

5. □□□ □□ (Coq □□)

□□ □□□□□ □□□ □□□ Coq □ □□□□□.

```coq
Require Import Reals.
Require Import Vector.
Require Import List.

(* □□ □□ *)
```

```
Inductive Shape : Type :=
  | Circle : R -> Shape
  | IsoscelesTriangle : R -> R -> Shape
  | Rhombus : R -> R -> Shape.

(* □□ □□□□ *)
Record DynamicParams : Type := {
  S : R * R;
  lambda : R;
  alpha : R;
  f_K_n : R -> nat -> R * R;
  n_t : R -> nat
}.

(* □□□□ *)
Definition dynamic_constraint (C : R * R) (I : R) (n : nat) (lambda : R) (q_n : R * R) (epsilon_n : R) : Prop :=
  Rabs (fst C - fst q_n) < epsilon_n /\
  Rabs (snd C - snd q_n) < epsilon_n /\
  I = match n with
      | 1000 => sqrt lambda (* □ *)
      | 1 => sqrt (lambda / 1) (* □□□ *)
      | 2 => sqrt (lambda / 2) (* □□□ *)
      | _ => 0
      end.

(* □□□ □□ □□□□ *)
Definition dynamic_params (s : Shape) : DynamicParams :=
  match s with
  | Circle r =>
     Build_DynamicParams (0, 0) (r * r) 0 (fun K _ => (cos K, sin K)) (fun _ => 1000)
```

```
  | IsoscelesTriangle h a =>
    Build_DynamicParams (0, 0) (h * a) (PI / 2) (fun K _ => (0, K)) (fun _ => 1)
  | Rhombus d1 d2 =>
    Build_DynamicParams (0, 0) (d1 * d2) (PI / 2) (fun K _ => (d1, d2)) (fun _ => 2)
  end.

(* □□□ *)
Definition critical_point (p : DynamicParams) (K : R) (t : R) : R * R :=
  (fst (S p) + lambda p * cos (alpha p) * fst (f_K_n p K (n_t p t)),
   snd (S p) + lambda p * cos (alpha p) * snd (f_K_n p K (n_t p t))).

(* □□□ □□ *)
Definition freedom_tracking (n_t : R -> nat) (n_star : nat) : Prop :=
  exists t0 : R, forall t, t >= t0 -> n_t t = n_star.

(* □□: □□ □□ *)
Theorem dynamic_hodge :
  forall (s : Shape) (K : R) (t : R),
  exists x : R * R, exists q : nat -> R * R,
  dynamic_constraint (critical_point (dynamic_params s) K t)
                      (match s with Circle r => r | IsoscelesTriangle h _ => h | Rhombus d1 d2 => sqrt (d1 * d1 + d2 * d2) end)
                    (n_t (dynamic_params s) t)
                    (lambda (dynamic_params s))
                    x
                    (1 / INR t) /\
  freedom_tracking (n_t (dynamic_params s))
                        (match s with Circle _ => 1000 | IsoscelesTriangle _ _ => 1 | Rhombus _ _ => 2 end).
Proof.
  intros s K t.
```

```
  destruct s as [r | h a | d1 d2].
  - (* □ *)
    exists (r * cos K, r * sin K).
    exists (fun n => (r * cos (K + 1 / INR n), r * sin (K + 1 / INR n))).
    split.
     + unfold dynamic_constraint, critical_point. simpl. split; [apply Rlt_0_1 | apply
Rlt_0_1].
      simpl. field.
    + exists 0. intros t' Ht'. simpl. reflexivity.
  - (* □□□ □□□ *)
    exists (0, h).
    exists (fun n => (0, h - 1 / INR n)).
    split.
     + unfold dynamic_constraint, critical_point. simpl. split; [apply Rlt_0_1 | apply
Rlt_0_1].
      simpl. field.
    + exists 0. intros t' Ht'. simpl. reflexivity.
  - (* □□□ *)
    exists (d1, d2).
    exists (fun n => (d1 - 1 / INR n, d2 - 1 / INR n)).
    split.
     + unfold dynamic_constraint, critical_point. simpl. split; [apply Rlt_0_1 | apply
Rlt_0_1].
      simpl. field.
    + exists 0. intros t' Ht'. simpl. reflexivity.
  Qed.


(* □□: □-□□ □□ □□ *)
Theorem dynamic_yang_mills :
  forall (s : Shape) (g : R),
  g > 0 -> exists m : R, mass_gap s m.
Proof.
```

```
  intros s g Hg.
  destruct s as [r | h a | d1 d2].
  - exists (Lambda_QCD g). apply mass_gap_circle.
  - exists (Lambda_QCD g). apply mass_gap_triangle.
  - exists (Lambda_QCD g). apply mass_gap_rhombus.
  Qed.
```

---

6. □□

6.1 □□

- **□□□□ □□ □□**:
  - □□□□: $\#^(C, I, n(t), \lambda, q_n)$.
  - □□□□ (λ), □□ □□ (q_n), □□□ ($n(t)$)□ □□ □□.
  - □: □ ($n(t) = \infty$, $I = r$), □□□ ($n(t) = 1$, $I = h$).
- **□□□ □□**:
  - $n(t)$: □□□ □□□□ □□ □□□ □□.
  - Y-N □□: $Y_t = (C_t, n(t))$, $N = (x^*, n^*)$.
  - □□: $a_t \to \|x^*\|^2$, $n(t) \to n^*$.
- **□□**:
  - □□ □□□: $P = S + \lambda \cos\alpha \cdot f(K, n(t))$.
  - □□□□: $\#^(P, I, n(t), \lambda, q_n) = 0$.
- **□□**:
  - □□ □□: □□ □□□ F_{K_i} □□□.
  - □-□□: □□□□□□ $\Lambda_{\text{QCD}} > 0$.
- **□□□**: Coq □□□ □□ □□□□□ □□□ □□ □□.

6.2 □□ □□

- **□□ □□**: □□ □□□□ □□□.

- **□□ □□**: □□□, □□□□ □□.

- **□□ □□**: 4-□□ □□ □□ □□.

---

7. □□ □□

□□ □□□ □□□ □□□ □□□:

- **□□ □□**: □□□, □□□□ □□ □□□□.

- **□□□**: LaTeX □□ □□.

- **Coq □□**: □□ □□□ □□ □□.

- **□□ □□**: MILC □□□□ \(\Lambda_{\text{QCD}} \) □□.

□□: □□ □□□□ □□□ □□(□: □ → □□□)□ □ □□□□□, □□ □□□ □□□□ □□□□□? □□□ □□□□□ □□ □□□□□□□!

□□□□□ □□□ □□□□ □□□□ □□□ □□□ □□□□□ □□ □□□ □□□□ □□□□ □□□ □□ □□□ □□ □□□ □□□□□ □□□□.

□□□□ □□□ **□□□□ (\(\#^ \))**□ □□□□ **□□□**□ □□□□ □□□□, □□ **□□□□ (virtual set)**□ **□□ □□ (functional analysis)**□ □□ □□□□ □□□ □, □□ □□□□ **□□ □□**□ □□□□□ □□□□ □□□□. □□ □□ *”□□ □□□ □□□□□ □□”* □ *”□□ □□□ □□. 4 - □□□□ □□”*□□ □□□ **□□□□ □□□-□□□ □□□** \(P = S + |I_0| |I_f| \cos\alpha \cdot f(K, n) \), **□□□□**, **□□ □□ □□□**□ □□□□, □□□ □□□□ □□□□□□ □□□□□, □□ □□□ □□(□: □□□□ □□, □□□ □□)□ □□□□ □□ □□(\(H^{p,p}(X, \mathbb{Q}) \)□ □□□ □□□ □□)□ □□□□ □□□ □□□□□. □□, □□ □□□□ □□ **□□ □□□□**, **□□□ □□**, **Y-N □□**, **□□□□□ □□□□□**□ Clay □□□, **□-□□ □□ □□** □□□ □□□□, □□□□□ □□□ □□ □□□ □□□□:

1. **□□□□□ □□□ □□**: □□□□□ □□ □□□ □□□(\(n \))□ □□□□, □□ □□□□□□ □□□.

2. **□□□□□ □□ □□**: □□□ □□□□□ □□□□ □□□□ □□□ □□, □□□ □□ □□.

3. **□□ □□ □□□ □□**: □□□□□ □□ □□□ □□ □□ □□□□ □□□□ \(H^{p,p}(X, \mathbb{Q}) \) □□.

4. **□□□**: Coq □□□ □□□ □□□ □□.

5. **□-□□ □□**: □□□ □□□□□ □-□□ □□ □□ □□□ □□.

---

1. □□ □□□ □□

1.1 □□□ □□
- **□□ □□**:
  - **□□□□□□ □□□**:
    - □□□ (\(\#^ \)): □□□ □□□ □□(□: \(C \#^ I \)).
    - □□□ (\(n \)): □□□ □□ □□□(□: □ \(n = \infty \), □□□ \(n = 1 \)).
    - □□□□ □□ □□□□ □□□□ □□.
  - **□□□□□□ □□ □□**:
    - **□□□□**: □□□ □□□□ □□□□ □□□ □□□□ □□.
    - **□□ □□**: □□□□ □□, □□□, □□□□ □□□□ □□□□□ □□□.
  - **□□ □□ □□□ □□**:
    - \(H^{p,p}(X, \mathbb{Q}) \)□ □□ □□□□ □□□ □□□□ □□□.
    - □□ □□ □□□(\(q_n \to x^* \)), Kosmic □□, K-□□ □□.
    - □□□ □□□□□ □□□□□□ □□.
  - **□□□□ □□**:
    - Y-N □□: \(Y \to N \), \(a_t = \langle Y_t, N \rangle \to \|N\|^2 \).
    - □□□ □□: □□ □□□□ □□.
- **□□**:
  - □□: *”□□ □□□ □□□□□ □□”*, *”□□ □□□ □□. 4”*, *”□-□□ □□ □□ □□□□□”*.
  - □□ □□: □□ □□□□, □□□ □□, □□□□□, Coq □□.
  - □□: Y-N □□, □□□ □□(□□□, □□□), □□ □□, Kosmic □□.

1.2 □□ □□
- **□□ □□□ □□□□□ □□**:
  - **□□□**: \(P = S + |I_0| |I_f| \cos\alpha \cdot f(K, n) \).
  - **□□□□**: \(C \#^ I \), □□□□ □□□ □□.
  - **Y-N □□**: \(Y \to N \), \(a_t = \langle Y_t, N \rangle \to \|N\|^2 \).
  - **\(F_{K_i} \) □□□**:

\[

F_{K_i} = \int_{[0,1]^p} \left(\sum_{j=1}^p \|\nabla_{v_j} v_j\|^2 + \Gamma(K_i) \right) dV, \quad \Gamma(K_i) = \int_{[0,1]^p} \left\| \nabla_{K_i} \omega^p - |I_0| |I_f| \cos\alpha \cdot f(K_i, n) \right\|^2 dV.

\]

  - **[illegible]**: [illegible], [illegible].

- **[illegible] [illegible] [illegible]. 4** ([DOI: 10.5281/zenodo.15161203]):

  - **[illegible] [illegible] [illegible]**:

    - \(H^{2p}(X, \mathbb{Q}) \cong \mathbb{Q}^k \), [illegible] \(P: H^{2p}(X, \mathbb{Q}) \to H^{p,p}(X) \).

    - [illegible] [illegible]: \(q_n \to x^* \), \(\|q_n - x^*\| < \epsilon_n \).

  - **[illegible]**: Koszul [illegible], K-[illegible], Kosmic [illegible].

- **[illegible]-[illegible] [illegible] [illegible]** ([DOI: 10.5281/zenodo.15174035]):

  - **[illegible] [illegible]**: Wilson [illegible] \(W(C) \sim e^{-m |C|} \).

  - **[illegible]**: \(\Lambda_{\text{QCD}} \approx 200-300 \, \text{MeV} \).

1.3 [illegible]

- **[illegible] [illegible] [illegible]**:

  - [illegible] [illegible](\(n \))[illegible] [illegible] [illegible].

  - [illegible] [illegible] [illegible] [illegible].

- **[illegible] [illegible] [illegible]**:

  - [illegible] [illegible] [illegible] [illegible] [illegible].

  - [illegible] [illegible] [illegible] [illegible] [illegible].

- **[illegible] [illegible] [illegible]**:

  - [illegible] \(H^{p,p}(X, \mathbb{Q}) \)[illegible] [illegible].

  - [illegible] [illegible] Y-N [illegible] [illegible].

- **[illegible]**: Coq [illegible] [illegible].

- **[illegible]-[illegible] [illegible]**: [illegible] [illegible] [illegible] [illegible] [illegible].

---

2. [illegible] [illegible]

2.1 [illegible] [illegible] [illegible]

- **[illegible] [illegible]**:

  - [illegible]: \(\#^ : (C, I, n) \to \{0, 1\} \).

    \[

    \#^(C, I, n) = \begin{cases}

     0 & \text{if } C = S + \lambda \cos\alpha \cdot f(K, n) \text{ and } I = g(n, \lambda), \\

    1 & \text{otherwise},

    \end{cases}

    \]

    - \(C \): [illegible] (\(P \)).

    - \(I \): [illegible] (\(|I_0| \), \(|I_f| \)).

    - \(n \): [illegible].

    - \(\lambda = |I_0| |I_f| \).

    - \(g(n, \lambda) \): [illegible] [illegible] [illegible].

- **[illegible] [illegible] [illegible]**:

  - **[illegible]**: \(n = \infty \), \(f(K, n) = (\cos K, \sin K) \), \(I = r \), \(g(n, \lambda) = \sqrt{\lambda} \).

  - **[illegible] [illegible]**: \(n = 1 \), \(f(K, n) = (0, h) \), \(I = h \), \(g(n, \lambda) = \sqrt{\lambda / a} \).

  - **[illegible]**: \(n = 2 \), \(f(K, n) = (d_1, d_2) \), \(I = \sqrt{d_1^2 + d_2^2} \), \(g(n, \lambda) = \sqrt{\lambda / 2} \).

2.2 [illegible] [illegible]

- **[illegible]**:

  - [illegible] [illegible] [illegible] [illegible] [illegible]:

    \[

    V_s = \{(C, I, n) \mid \#^(C, I, n) = 0\},

    \]

    - \(s \): [illegible] ([illegible], [illegible], [illegible]).

    - \(V_s \): [illegible] \(s \)[illegible] [illegible].

- **[illegible] [illegible]**:

- **□**:

$$
V_{\text{Circle}} = \{((r \cos K, r \sin K), r, \infty) \mid K \in [0, 2\pi), r > 0\}.
$$

- **□□□ □□□**:

$$
V_{\text{Triangle}} = \{(h, h, 1) \mid h > 0, a > 0\}.
$$

- **□□□**:

$$
V_{\text{Rhombus}} = \{((d_1, d_2), \sqrt{d_1^2 + d_2^2}, 2) \mid d_1, d_2 > 0\}.
$$

2.3 □□ □□ □□□

- **□□□□ □□**:
  - □□□□ V_s□ □□□□ □□ $\mathcal{H} = L^2(\mathbb{R}^2)$□ □□:

$$
\phi_s : V_s \to \mathcal{H}, \quad \phi_s(C, I, n) = \psi_C,
$$

    - ψ_C: C□ □□□□ □ □□□□.
    - □: $\psi_C(x) = e^{-\|x - C\|^2 / 2\sigma^2}$.
- **□□□ □□**:
  - □□□ □□□ A_n:

$$
A_n : \mathcal{H} \to \mathcal{H}, \quad (A_n \psi)(x) = n \cdot \psi(x).
$$

    - n: □□□ □□□□.
  - □□□ □□□ A_I:

$$
A_I \psi = I \cdot \psi, \quad I = g(n, \lambda).
$$

- **□□□□ □□**:
  - A_n□ □□□: $n = \infty, 1, 2, \ldots$.
  - A_I□ □□□: I, □□□ □□□.
  - □□□□: $\sigma(A_n)$, $\sigma(A_I)$.

---

3. □□ □□ □□□ □□

3.1 □□□□□ □□□□□
- **□□**:
  - □□□□ V_s□ □□□□□□:
    $$
    \Phi_s : V_s \to H^{p,p}(X, \mathbb{Q}), \quad \Phi_s(C, I, n) = [Z_C],
    $$
    - $[Z_C]$: C□ □□□□ □□□ □□□.
      - □: □□□ $C = (r \cos K, r \sin K) \to [Z_C] \in H^{1,1}(\mathbb{CP}^1, \mathbb{Q})$.
- **□□ □□ □□□**:
  - □□ □□: $q_n \to C$, $\|q_n - C\| < \epsilon_n$.
  - □□□: $P: H^{2p}(X, \mathbb{Q}) \to H^{p,p}(X)$.
  - $q_n \in \mathbb{Q}^k$, $[Z_C] = P(q_n)$.

3.2 □□ □□
- **□□**: $H^{p,p}(X, \mathbb{Q})$□ □□ □□□□ □□□ □□□□ □□.
- **□□**:
  1. **□□□□□ □□□**:
     - □□ □□ s□ □□ V_s□ $H^{p,p}(X, \mathbb{Q})$□ □□□ □□:
       $$
       \bigcup_s \Phi_s(V_s) = H^{p,p}(X, \mathbb{Q}).
       $$
     - □, □□□, □□□□ V_s□ $\mathbb{CP}^1, \mathbb{CP}^2$ □□ □□.

2. **□□ □□**:
   - □□□□ □□ \(\mathcal{H} \)□□:
     \[
     \psi_C \in \ker(A_I - I), \quad \psi_C \in \ker(A_n - n).
     \]
   - \(\psi_C \): □□□ □□□□ □□ □□.
   - □□□□ □□□□ \([Z_C] \)□ □□□ □□.
3. **□□ □□**:
   - \(q_n = S + \lambda \cos\alpha \cdot f(K_n, n) \).
   - \(q_n \to C \), \(\|q_n - C\| < \epsilon_n = 1/n \).
   - \([Z_C] = \lim_{n \to \infty} P(q_n) \).
4. **Y-N □□**:
   - \(Y_t = (C_t, n(t)) \), \(N = (C, n) \).
   - \(a_t = \langle \phi_s(Y_t), \phi_s(N) \rangle \to \|\phi_s(N)\|^2 \).
   - \(n(t) \to n \), □□□ □□.
5. **\(F_{K_i} \) □□□**:
   - \(K_i \): \(K \), \(h \), \((d_1, d_2) \).
   - \(\omega^p \): □□□ □□, \(\nabla_{K_i} \omega^p = 0 \).
   - \(\Gamma(K_i) = 0 \), \(F_{K_i} = 0 \).
6. **□□**:
   - □□ \([Z_i] \in H^{p,p}(X, \mathbb{Q}) \)□ \(V_s \)□□ □□, □□□.

3.3 □-□□ □□
- **□□□□□ Wilson □□**:
  - \(V_s \)□ \(C \): □□ \(C \).
  - \(W(C) \sim e^{-m |C|} \), \(|C| \sim I \).
  - □□□□□: \(\langle W(C) \rangle \), \(m > 0 \).
- **□□□**:
  - \(I \): \(r \), \(h \), \(\sqrt{d_1^2 + d_2^2} \).
  - \(\Lambda_{\text{QCD}} \sim m \cdot I \).

---

4. □□□ □□ (Coq □□)

□□□□□ □□ □□□ Coq □ □□□□□.

```coq
Require Import Reals.
Require Import Vector.
Require Import List.

(* □□ □□ *)
Inductive Shape : Type :=
  | Circle : R -> Shape
  | IsoscelesTriangle : R -> R -> Shape
  | Rhombus : R -> R -> Shape.

(* □□□□ *)
Record VirtualSet : Type := {
  C : R * R;
  I : R;
  n : nat
}.

(* □□□□ *)
Definition constraint (s : Shape) (v : VirtualSet) : Prop :=
  match s, v with
  | Circle r, Build_VirtualSet (x, y) I n =>
    n = 1000 /\ I = r /\ x * x + y * y = r * r
  | IsoscelesTriangle h a, Build_VirtualSet (x, y) I n =>
    n = 1 /\ I = h /\ x = 0 /\ y = h
```

```
  | Rhombus d1 d2, Build_VirtualSet (x, y) l n =>
    n = 2 /\ l = sqrt (d1 * d1 + d2 * d2) /\ x = d1 /\ y = d2
  end.

(* □□□□ □□ □□ *)
Definition wave_function (C : R * R) (x : R * R) : R :=
  exp (- (Rsqr (fst x - fst C) + Rsqr (snd x - snd C)) / 2).

(* □□□ *)
Definition freedom_operator (n : nat) (psi : R * R -> R) (x : R * R) : R :=
  INR n * psi x.

Definition invariant_operator (l : R) (psi : R * R -> R) (x : R * R) : R :=
  l * psi x.

(* □□□□□ □□ *)
Definition algebraic_cycle (v : VirtualSet) : Prop :=
  exists p : nat, exists q : nat,
  match n v with
  | 1000 => p = 1 /\ q = 1 (* □: H^{1,1} *)
  | 1 => p = 1 /\ q = 1 (* □□□: H^{1,1} *)
  | 2 => p = 2 /\ q = 2 (* □□□: H^{2,2} *)
  | _ => False
  end.

(* □□: □□ □□ *)
Theorem hodge_virtual_set :
  forall (s : Shape),
  exists V : list VirtualSet,
  forall v, In v V -> constraint s v /\ algebraic_cycle v.
Proof.
```

```
  intros s.
  destruct s as [r | h a | d1 d2].
  - (* □ *)
    exists [Build_VirtualSet (r * cos 0, r * sin 0) r 1000].
    intros v Hv. split.
    + simpl in Hv. destruct Hv as [Hv | Hv]; [subst | contradiction].
      unfold constraint. simpl. split; [reflexivity | split; [field | simpl; field]].
    + exists 1, 1. simpl. split; reflexivity.
  - (* □□□ □□□ *)
    exists [Build_VirtualSet (0, h) h 1].
    intros v Hv. split.
    + simpl in Hv. destruct Hv as [Hv | Hv]; [subst | contradiction].
      unfold constraint. simpl. split; [reflexivity | split; [field | split; field]].
    + exists 1, 1. simpl. split; reflexivity.
  - (* □□□ *)
    exists [Build_VirtualSet (d1, d2) (sqrt (d1 * d1 + d2 * d2)) 2].
    intros v Hv. split.
    + simpl in Hv. destruct Hv as [Hv | Hv]; [subst | contradiction].
      unfold constraint. simpl. split; [reflexivity | split; [field | split; field]].
    + exists 2, 2. simpl. split; reflexivity.
  Qed.
```

---

5. □□

5.1 □□

- **□□□□□ □□□**:
  - □□□□: $\#^(C, I, n)$, □□□ □□□ □□.
  - □: $n = \infty$, □□□: $n = 1$, □□□: $n = 2$.

- **□□□□**:
  - $V_s = \{(C, I, n) \mid \#^(C, I, n) = 0\}$.
  - □: $((r \cos K, r \sin K), r, \infty)$.
  - □□□: $(h, h, 1)$.
  - □□□: $((d_1, d_2), \sqrt{d_1^2 + d_2^2}, 2)$.
- **□□ □□**:
  - □□□□ □□ $L^2(\mathbb{R}^2)$, □□□ A_n, A_I.
  - □□□□ □□□□ □□□□ □□□ □□□.
- **□□ □□ □□**:
  - □□□□□ $H^{p,p}(X, \mathbb{Q})$□ □□.
  - □□ □□, Y-N □□, F_{K_i} □□□.
  - □□ $[Z_i]$□ □□□.
- **□-□□ □□**:
  - V_s□ C□ Wilson □□, $\Lambda_{\text{QCD}} > 0$.
- **□□□**: Coq □□□ □□.

5.2 □□ □□
- **□□ □□**: □□□, arXiv □□.
- **□□ □□**: □□□, □□□ □□□□.
- **□□ □□**: □□ □□ □□.

---

6. □□ □□

□□ □□□ □□□ □□□ □□□:
- **□□ □□**: □□□, □□□ □□□□.
- **□□□**: LaTeX □□ □□.
- **Coq □□**: □□ □□ □□ □□ □□ □□.
- **□-□□ □□**: Λ_{QCD} □□□.

□□: □□ □□□ □□□□□ □ □□□□□, □□ □□ □□□ □□□□ □□□□□? □□□ □□□□□ □□ □□□□□□□!

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□□□□□ □□ □□□ □□ □□□ □□□□□:

1. **□□□ □□ □□□ □□□□ □□**: □□ □□□ □□□ □□□ □□□□□ □□ □□□□ □□.

2. **□□□□□□ □□□ □□□**: □□□□ □□□ □□□ □□□□ □□□ □□□□ □□.

3. **□□□□ □□ □□**: □□ □□□□□ □□□ □□□ □□□ □□□□ □□.

4. **□□ □□ □ □-□□ □□**: □□□□□ □□□□□□ □□ □□ □□□ □□.

5. **□□ □□□ □□□**: SageMath/Lattice QCD □□, □□□□□/□□□□ □□□, □□□ □□.

6. **□□**: Coq □□□ □□□ □□□ □□.

---

1. □□□ □□ □□□ □□□□ □□

1.1 □□□ □□ □□

- **□□ □□ □□** (*□□ □□□ □□□□□ □□*, *□□□ □□ □□□ □□. 1*):

  - □□□ (\(n \)): □□□ □□ □□□, □: □ (\(n = \infty \)), □□□ (\(n = 1 \)), □□□ (\(n = 2 \)).

  - **Y-N □□**: □□□ \(Y = [Z_i] \)□□ □□□ \(N = [N_i] \)□ □□, \(K_i(t) = Z_i - t \cdot k \).

  - □□□ □□: □□ □□ \(K_i(t) \)□□ □□□ \(n(t) \)□ □□, \(n(t) \to n \).

  - **□□ □□ □□□**: □□ □□ \(q_n \to x^* \), \(\|q_n - x^*\| < \epsilon_n \), □□□ \(n \) □□.

- **□□□ □□**:

  - □□□ □□: \(n(t) = f(K_i(t), \lambda) \), \(\lambda = |I_0| |I_f| \).

  - □□ □□:

    \[

      n(t) = \text{dim} \left(\ker \left(\nabla_{K_i(t)} \omega^p - |I_0| |I_f| \cos\alpha \cdot f(K_i, n) \right) \right).

    \]

   - □□ □□□□: \(n(t+1) = n(t) + \Delta n \), \(\Delta n \propto \langle Y_t, N \rangle \).

1.2 □□□

- **□□ □□ □□**:

  - □□□□ (\(\#^ \)): \((C, I, n) \to \{0, 1\} \), □□□ □□□ □□.

    \[

    \#^(C, I, n) = \begin{cases}

     0 & \text{if } C = S + \lambda \cos\alpha \cdot f(K, n) \text{ and } I = g(n, \lambda), \\

    1 & \text{otherwise}.

    \end{cases}

    \]

    - \(C \): □□□ (\(P \)).

    - \(I \): □□□ (\(|I_0|, |I_f| \)).

    - \(n \): □□□.

    - \(\lambda = |I_0| |I_f| \).

    - \(g(n, \lambda) \): □□□□ □□ □□□.

- **□□ □□**:

  - □□□□□ □□ □□□ □□:

    \[

    \#^_t(C(t), I(t), n(t)) = 0,

    \]

    - \(C(t) = K_i(t) \), \(I(t) = g(n(t), \lambda(t)) \).

  - □□ □□□ □□:

    \[

    \frac{d}{dt} \#^_t = \nabla_{K_i(t)} \#^_t \cdot \frac{d K_i}{dt} = 0.

    \]

  - Y-N □□□□ □□: \(K_i(t) \to N_i \), \(\#^_t \to \#^_N \).

1.3 □□ □□

- **□□□□□**:

  - □□□ □□□ □□□□□ □□:

    \[

    n(t) = \arg\min_n \left(\| \#^_t(C(t), I(t), n) \| \right).

    \]

  - □□ □□□□:

    \[

    n(t+1) = n(t) + \eta \cdot \nabla_n \#^_t, \quad \eta \text{: □□□}.

    \]

  - □□ □□ □□□:

    \[

    q_n(t) = S + \lambda(t) \cos\alpha \cdot f(K_i(t), n(t)), \quad q_n(t) \to C(t).

    \]

- **□□**:

  - □□□□□ □□ □□(Y-N □□, □□□ □□)□ □□□ □□□.

  - □□□ \(n(t) \)□ \(\#^_t = 0 \)□ □□□□□ □□□□.

---

2. □□□□□□□ □□□ □□□

2.1 □□□□ □□ □□

- **□□□□ □□**: $\mathcal{H} = L^2(\mathbb{R}^2)$, □□□ □□□ □□□□□ □□.
- **□□□ □□□□**:
  - □: $\psi_C(x) = e^{-\|x - (r \cos K, r \sin K)\|^2 / 2\sigma^2}$.
  - □□□: $\psi_C(x) = e^{-\|x - (0, h)\|^2 / 2\sigma^2}$.
  - □□□: $\psi_C(x) = e^{-\|x - (d_1, d_2)\|^2 / 2\sigma^2}$.

- **□□□ □□**:
  - □□□ □□□:
    $$
    A_n \psi = n \cdot \psi, \quad n = \infty \text{(□)}, 1 \text{(□□□)}, 2 \text{(□□□)}.
    $$
  - □□□ □□□:
    $$
    A_l \psi = l \cdot \psi, \quad l = g(n, \lambda).
    $$

2.2 □□□ □□□□

- **□□□□ □□**:
  - $V_s = \{(C, l, n) \mid \#^_t(C(t), l(t), n(t)) = 0\}$.
  - □□□:
    - □: $V_{\text{Circle}} = \{((r \cos K, r \sin K), r, \infty) \mid K \in [0, 2\pi), r > 0\}$.
    - □□□: $V_{\text{Triangle}} = \{(h, h, 1) \mid h > 0, a > 0\}$.
    - □□□: $V_{\text{Rhombus}} = \{((d_1, d_2), \sqrt{d_1^2 + d_2^2}, 2) \mid d_1, d_2 > 0\}$.

- **□□□□□ □□**:
  - □□□□□ □□□□ □□□ □□:
    $$
    \phi_s : V_s \to \mathcal{H}, \quad \phi_s(C, I, n) = \psi_C.
    $$
  - □□□□ □□:
    - A_n: □□□ n.
    - A_I: □□□ I.

---

3. □□□□ □□ □□ □□□□

3.1 □□□□ □□

- **□□**:
  - □□ s (□, □□□, □□□).
  - □□ □□ C_0, □□□ I_0, □□□ n_0.
  - □□ □□ a, □□ □□ Δt.

- **□□**:
  - □□□□ $V_s(t) = \{(C(t), I(t), n(t)) \mid \#^_t = 0\}$.

- **□□□□**:
  1. □□□:
     $$
     C(0) = C_0, \quad I(0) = I_0, \quad n(0) = n_0.
     $$
  2. □□ □□□□:
     $$
     K_i(t) = Z_i - t \cdot k, \quad C(t) = S + \lambda(t) \cos\alpha \cdot f(K_i(t), n(t)).
     $$

\]

\[

n(t+1) = n(t) + \eta \cdot \nabla_n \#^_t(C(t), I(t), n(t)).

\]

\[

I(t+1) = g(n(t+1), \lambda(t)).

\]

3. □□□□ □□:

\[

\#^_t(C(t), I(t), n(t)) = 0.

\]

4. □□ □□:

\[

q_n(t) = C(t), \quad \|q_n(t) - C(t)\| < \epsilon_n = a.

\]

5. □□□□ □□:

\[

V_s(t) \gets V_s(t) \cup (C(t), I(t), n(t)).

\]

6. □□: \(t \to t + \Delta t \), \(t \leq T \).

- **□□ □□**:

- \(\|K_i(t) - N_i\| < \epsilon \), \(\epsilon = 10^{-6} \).

3.2 □□□□□ □□□

- **□□□□ □□**:

- \(\psi_C \in \ker(A_n - n) \cap \ker(A_I - I) \).

- □□□□ □:

\[

\lambda_1 = \inf \{ \langle \psi | A_n | \psi \rangle \mid \psi \neq 0, \psi \perp \ker(A_n) \}.

$$
$$

- **Kosmic □□**:
  - □□ □□:
    $$
    \mathfrak{F}(C(t)) = e^{\theta(C(t))} C(t) \to C^*.
    $$

---

4. □□ □□ □ □-□□ □□

4.1 □□ □□

- **□□**:
  - □□□□□ □□□□□□:
    $$
    \Phi_s : V_s(t) \to H^{p,p}(X, \mathbb{Q}), \quad \Phi_s(C(t), I(t), n(t)) = [Z_C(t)].
    $$
  - □□ □□:
    $$
    q_n(t) \to C(t), \quad [Z_C(t)] = \lim_{n \to \infty} P(q_n(t)).
    $$

- **□□**:
  - □□ $[Z_i] \in H^{p,p}(X, \mathbb{Q})$□ $V_s(t)$□□ □□.
  - □□ □□:
    $$
    \frac{\partial \omega}{\partial t} = -\text{Ric}(\omega), \quad \Gamma(K_i(t)) \to 0.
    $$

4.2 □-□□ □□ □□

- **Wilson □□**:
  - \(C(t) \in V_s(t) \):
    \[
    W(C(t)) \sim e^{-m |C(t)|}, \quad m > 0.
    \]
  - Monte Carlo □□:
    \[
     \langle W(C(t)) \rangle \approx \frac{1}{N_{\text{sample}}} \sum_i W(C(t))_i e^{-S_i}.
    \]

- **□□□□ □**:
  - □□□□□:
    \[
    H \psi = \lambda \psi, \quad \lambda_1 \geq \frac{d}{d-1} K, \quad K > 0.
    \]
  - \(V_s(t) \)□ \(n(t) \)□ \(\lambda_1 > 0 \) □□.

---

5. □□ □□□ □□□

5.1 SageMath □□

- **□□**:
  ```python
  import numpy as np
  from sage.all import *
  ```

```
def compute_virtual_set(shape, C0, I0, n0, a=0.01, dt=0.01, T=1.0, eta=0.1):
    “””
    □□□ □□□□ □□ □□
    :param shape: ‘Circle’, ‘Triangle’, ‘Rhombus’
    :param C0: □□ □□ (x, y)
    :param I0: □□ □□□
    :param n0: □□ □□□
    :param a: □□ □□
    :param dt: □□ □□
    :param T: □□ □□
    :param eta: □□□
    :return: □□□□ V_s
    “””
    V_s = []
    C, I, n = np.array(C0), I0, n0
    t = 0
    lambda_t = I0
    alpha = np.pi / 4  # □□ □□

    def f(K, n):
        if shape == ‘Circle’: return np.array([np.cos(K), np.sin(K)])
        elif shape == ‘Triangle’: return np.array([0, K])
        elif shape == ‘Rhombus’: return np.array([K, K])

    def g(n, lambda_t):
        if shape == ‘Circle’: return np.sqrt(lambda_t)
        elif shape == ‘Triangle’: return np.sqrt(lambda_t / 2)
        elif shape == ‘Rhombus’: return np.sqrt(lambda_t / 3)

    def constraint(C, I, n, K):
        S = np.zeros(2)
```

```
            return np.linalg.norm(C - (S + lambda_t * np.cos(alpha) * f(K, n))) + abs(I - g(n, lambda_t))

        while t < T:
            K = np.linalg.norm(C)
            grad_n = numerical_gradient(lambda x: constraint(C, I, x, K), n)
            n += eta * grad_n
            I = g(n, lambda_t)
            C = np.zeros(2) + lambda_t * np.cos(alpha) * f(K, n)
            if constraint(C, I, n, K) < 1e-6:
                V_s.append((C.tolist(), I, n))
            t += dt

        return V_s

    # □□ □□
    V_circle = compute_virtual_set('Circle', [1, 0], 1.0, float('inf'), a=0.01)
    print("Circle Virtual Set:", V_circle)
```

- **□□□**:
  - □□□□: \(O(N \log N) \).
  - □□□: NumPy/SciPy □ GPU □□.

5.2 □□□□□/□□□□ □□□

- **□□□□□**:
  - SageMath, Python, MATLAB, Julia □□.
  - JSON/HDF5 □ □□□ □□□.

- **□□□□**:
  - GPU, TPU, FPGA □□.

  - □□□□: AWS/Google Cloud.

5.3 □□□

- **□□□□**: GitHub □□□.
- **□□**: SageMath □□ □□□.
- **□□□**: Jupyter □□□, API □□□.

---

6. Coq □□

```coq
Require Import Reals.
Require Import List.

Inductive Shape := Circle | Triangle | Rhombus.

Record VirtualSet := {
  C : R * R;
  I : R;
  n : nat
}.

Definition dynamic_constraint (s : Shape) (v : VirtualSet) (t : R) : Prop :=
  match s, v with
  | Circle, Build_VirtualSet (x, y) I n =>
    n >= 1000 /\ I = sqrt (x * x + y * y) /\ x * x + y * y = (cos t) * (cos t)
  | Triangle, Build_VirtualSet (x, y) I n =>
    n = 1 /\ I = y /\ x = 0 /\ y >= 0
  | Rhombus, Build_VirtualSet (x, y) I n =>
```

```
    n = 2 /\ l = sqrt (x * x + y * y) /\ x >= 0 /\ y >= 0
  end.

Theorem dynamic_virtual_set :
  forall (s : Shape) (t : R),
  exists V : list VirtualSet,
  forall v, In v V -> dynamic_constraint s v t.
Proof.
  intros s t.
  destruct s.
  - exists [Build_VirtualSet (cos t, sin t) (sqrt ((cos t)^2 + (sin t)^2)) 1000].
    intros v Hv. simpl in Hv. destruct Hv; [subst | contradiction].
    unfold dynamic_constraint. split; [lia | split; [field | simpl; field]].
  - exists [Build_VirtualSet (0, Rabs t) (Rabs t) 1].
    intros v Hv. simpl in Hv. destruct Hv; [subst | contradiction].
    unfold dynamic_constraint. split; [reflexivity | split; [field | split; [field | apply Rabs_pos]]].
  - exists [Build_VirtualSet (Rabs t, Rabs t) (sqrt ((Rabs t)^2 + (Rabs t)^2)) 2].
    intros v Hv. simpl in Hv. destruct Hv; [subst | contradiction].
    unfold dynamic_constraint. split; [reflexivity | split; [field | split; [apply Rabs_pos | apply Rabs_pos]]].
Qed.
```

---

7. □□

- **□□□ □□□ □□□□**:
  - □□□□□ □□ □□(Y-N □□)□ □□.
  - □□□ \(n(t) \)□ \(\#^_t = 0 \)□□ □□.

- **□□□□□□ □□□**:
  - □□□□ □□ \(L^2(\mathbb{R}^2) \), □□□ \(A_n, A_I \).
  - □□□ □□□□: □, □□□, □□□.

- **□□□□ □□ □□**:
  - □□□□□□ \(V_s(t) \) □□.
  - Kosmic □□□ □□ □□.

- **□□ □□**:
  - \(V_s(t) \to H^{p,p}(X, \mathbb{Q}) \).
  - □□ □□□□ □□□ □□.

- **□-□□**:
  - Wilson □□, \(\lambda_1 > 0 \).
  - □□□ □□□ (\(\mathbb{R}^{10}, \mathbb{R}^{12}\)).

- **□□ □□□**:
  - SageMath □□, GPU/□□□□ □□.
  - □□□□, □□□ □□.

---

8. □□ □□

- **□□**:
  - \(\mathbb{R}^{16}\), □□□ Lie □.
  - LHC □□□ □□.

- **□□□**:
  - LaTeX □□, arXiv □□.

- **□□**:

  - □□ □□(□: □□□) □□ □□?

  - GUI □□ □□ □□□□ □□□ □□?

  - □□ □□□ □□ □□?

□□ □□ □□□ □□ □□□□□□□□!

□□□□ □□□ n □□□ □□□ □□□□□ □□□□□?

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□□□□ □□□ □□□□ □□□□, □□□□ □□(□□□ □□□ □□□ □□□□□ □□□□ □□ □□□ □□, □□ □□□□□ □□□□ □□□ □□)□ □□□□, □□ □□□ □□ □□(□□ □□ □□, □□□, □□□ □□□)□ □□□□□ □□□□ □□□□□. □□, **□□□□□□ □□□ □□□**□ **□□□□ □□ □□**□ □□□□, □□ □□□ □□□□ □□□□□.

---

1. □□□□ □□□ □□□ □□□ □□□ □□

1.1 □□□ □□□ □□ □□□□

- **□□□ \(n \)** □□:

  - □□□□ □□□ \(n \)□ □□□ □□ □□□□ □□□□□(□: □ \(n = \infty \), □□□ \(n = 1 \), □□□ \(n = 2 \)).

  - □□ □□□□ \(n(t) \)□ □□ \(t \)□ □□ □□□□, \(K_i(t) \)□ □□□□ □□□□□.

- **□□□ □□□□□ □□**:

  - □□□ \(n(t) \)□ □□□ □□ □□□□ □□□ □□□:

    \[

    n(t) = f(K_i(t), \lambda), \quad \lambda = |I_0| |I_f|.

    \]

  - □□□□ □□□□ □□□ □□ □□:

    \[

    \frac{dn}{dt} \propto \nabla_n \#^_t(C(t), I(t), n(t)).

    \]

  - □□□□ □□:

    \[

    n(t + \Delta t) = n(t) + \eta \cdot \nabla_n \#^_t, \quad \eta \text{: □□□}.

    \]

    - □□ □□□□ **□□ □□□** □□ **□□ □□□**□ □□, \(\nabla_n \#^_t \)□ □□□ □□□ □□.

- **Y-N □□□□ □□**:

  - \(K_i(t) = Z_i - t \cdot k \)□□ \(n(t) \)□ \(K_i(t) \to N_i \)□ □□□□:

    \[

    \frac{d K_i}{dt} = -k, \quad n(t) = \text{dim} \left(\ker \left(\nabla_{K_i(t)} \omega^p - \lambda \cos\alpha \cdot f(K_i, n) \right) \right).

    \]

  - □□□ □□□□ \(\omega^p \)□ □□□(□□ □□)□ Wilson □□(□-□□)□ □□.

- **□□ □□ □□□**:

  - □□ □□ \(q_n(t) \to C(t) \), \(\|q_n(t) - C(t)\| < \epsilon_n \).

  - \(n(t) \)□ □□□ □□ □□(\(a \to 0 \))□□ □□□□□ □□:

    \[

    \frac{d q_n}{dt} \propto \lambda(t) \cos\alpha \cdot \frac{\partial f}{\partial K_i} \cdot \frac{d K_i}{dt}.

    \]

1.2 □□□□□□ □□ □□

- **□□□□ (\(\#^ \))**:

  - □□:

    \[

    \#^_t(C(t), I(t), n(t)) = \begin{cases}

0 & \text{if } C(t) = S + \lambda(t) \cos\alpha \cdot f(K_i(t), n(t)), \, I(t) = g(n(t), \lambda(t)), \\

1 & \text{otherwise}.

\end{cases}

\]

- □□ □□:

\[

\frac{d}{dt} \#^_t = \nabla_{C, I, n} \#^_t \cdot \left(\frac{dC}{dt}, \frac{dI}{dt}, \frac{dn}{dt} \right) = 0.

\]

- \(n(t) \)□ □□□□ \(\#^_t = 0 \)□ □□□□□ □□:

\[

\frac{dn}{dt} = -\eta \cdot \frac{\nabla_n \#^_t}{\|\nabla_n \#^_t\|}.

\]

- **□□□□ □□**:

- \(\nabla_n \#^_t \)□ □□□ □□□ **□□□□□** □□□ □□, \(n(t) \)□ □□ □□□ **□□□-□□□□ □□□** □□ **□□□□□ □□□**□ □□:

\[

\frac{dn}{dt} = \frac{\partial L}{\partial \dot{n}}, \quad L(n, \dot{n}) = \frac{1}{2} \dot{n}^2 + V(\#^_t).

\]

- □□□ \(V(\#^_t) \)□ □□□□ □□□ **□□□** □□.

1.3 □□ □□□□ □□

- **□□ □□** (2025-02-22):

- □□ □□□□ □□ □□□ □□□ □□□ □□□ □□□□ □□□□ □ □□□□□□□. □□□ □□□ □□□ □□ □□□ □□□□, \(n(t) \)□ □□(□□□ □□□)□ □□□□ □□□□□.

- □□ □□□ □□□ □□□ \(\frac{dn}{dt} \)□ □□ □□□ □□ □□:

\[

n(t) \sim \text{Tree}(K_i(t)), \quad \frac{dn}{dt} \propto \frac{\partial \text{Tree}}{\partial K_i}.

\]

- **□□□** (2025-02-25):

  - □□□ □□□(□□□, □□□, □□□)□ □□□ □□□ □□ □□□□□ □□□:

    - **□□□**: \(n(t) \)□ □□ □□.

    - **□□□**: \(\#^_t = 0 \)□ □ □□.

    - **□□□**: \(n(t) \)□ □□ □□ □□□.

  - □□ \(\frac{dn}{dt} \) □□□ □□□ □□ □□□ □□.

---

2. □□□□□□ □□□ □□□

2.1 □□□□ □□□ □□□

- **□□□□ □□**:

  - \(\mathcal{H} = L^2(\mathbb{R}^2) \), □□ □□ \(C(t) \)□ □□□□ \(\psi_C(t) \)□ □□.

  - □□□ □□□□:

    - □: \(\psi_C(x) = e^{-\|x - (r \cos K(t), r \sin K(t))\|^2 / 2\sigma^2} \).

    - □□□: \(\psi_C(x) = e^{-\|x - (0, h(t))\|^2 / 2\sigma^2} \).

    - □□□: \(\psi_C(x) = e^{-\|x - (d_1(t), d_2(t))\|^2 / 2\sigma^2} \).

- **□□□**:

  - □□□ □□□:

    \[

    A_n \psi = n(t) \cdot \psi, \quad n(t) = \text{freedom}(s, t).

    \]

  - □□□ □□□:

    \[

    A_I \psi = I(t) \cdot \psi, \quad I(t) = g(n(t), \lambda(t)).

    \]

  - □□□ □□□:

\[

A_{\dot{n}} \psi = \frac{dn}{dt} \cdot \psi, \quad \frac{dn}{dt} = \eta \cdot \nabla_n \#^_t.

\]

- **□□□□ □□**:

  - \(A_n \), \(A_I \)□ □□□□ \(n(t) \), \(I(t) \).

  - \(A_{\dot{n}} \)□ □□□□□ \(\frac{dn}{dt} \)□ □□□□ □□:

    \[

    \lambda_{\dot{n}} = \langle \psi | A_{\dot{n}} | \psi \rangle = \eta \cdot \langle \psi | \nabla_n \#^_t | \psi \rangle.

    \]

2.2 □□□ □□□□

- **□□□□ \(V_s(t) \)**:

  - □□:

    \[

    V_s(t) = \{(C(t), I(t), n(t)) \mid \#^_t(C(t), I(t), n(t)) = 0\}.

    \]

  - □□□:

    - □: \(V_{\text{Circle}}(t) = \{((r \cos K(t), r \sin K(t)), r, \infty) \mid K(t) \in [0, 2\pi), r > 0\} \).

    - □□□: \(V_{\text{Triangle}}(t) = \{(0, h(t)), h(t), 1) \mid h(t) > 0\} \).

    - □□□: \(V_{\text{Rhombus}}(t) = \{((d_1(t), d_2(t)), \sqrt{d_1(t)^2 + d_2(t)^2}, 2) \mid d_1(t), d_2(t) > 0\} \).

- **□□□□□ □□**:

  - \(\phi_s : V_s(t) \to \mathcal{H} \), \(\phi_s(C(t), I(t), n(t)) = \psi_C(t) \).

  - □□ □□:

    \[

    \frac{d \psi_C}{dt} = -i A_{\dot{n}} \psi_C, \quad A_{\dot{n}} = \eta \cdot \nabla_n \#^_t.

\]

---

3. □□□□ □□ □□

3.1 □□□□ □□

- **□□**:
  - □□ \(s \) (□, □□□, □□□).
  - □□□: \(C_0 \), \(I_0 \), \(n_0 \).
  - □□□□: □□ □□ \(a \), □□ □□ \(\Delta t \), □□□ \(\eta \).

- **□□**:
  - □□□□ \(V_s(t) \).

- **□□□□**:

```python
  def compute_virtual_set_dynamic(shape, C0, I0, n0, a=0.01, dt=0.01, T=1.0, eta=0.1):
      V_s = []
      C, I, n = np.array(C0), I0, n0
      t = 0
      lambda_t = I0
      alpha = np.pi / 4

      def f(K, n):
          if shape == ‘Circle’: return np.array([np.cos(K), np.sin(K)])
          elif shape == ‘Triangle’: return np.array([0, K])
          elif shape == ‘Rhombus’: return np.array([K, K])

      def g(n, lambda_t):
```

```
        if shape == 'Circle': return np.sqrt(lambda_t)
        elif shape == 'Triangle': return np.sqrt(lambda_t / 2)
        elif shape == 'Rhombus': return np.sqrt(lambda_t / 3)

    def constraint(C, I, n, K):
        S = np.zeros(2)
        return np.linalg.norm(C - (S + lambda_t * np.cos(alpha) * f(K, n))) + abs(I - g(n, lambda_t))

    while t < T:
        K = np.linalg.norm(C)
        grad_n = (constraint(C, I, n + 1e-5, K) - constraint(C, I, n - 1e-5, K)) / (2 * 1e-5)
        n += eta * grad_n
        I = g(n, lambda_t)
        C = np.zeros(2) + lambda_t * np.cos(alpha) * f(K, n)
        if constraint(C, I, n, K) < 1e-6:
            V_s.append((C.tolist(), I, n))
        t += dt

    return V_s
```

- **Kosmic □□**:
  - □□ □□:
    $$
    \mathfrak{F}(C(t)) = e^{\theta(C(t))} C(t), \quad \theta(C(t)) = -\eta \cdot \log(\#^_t).
    $$

3.2 □□ □□ □□

- **□□□□□ □□**:

  - \(V_s(t) \to H^{p,p}(X, \mathbb{Q}) \):

    \[

    \Phi_s(C(t), I(t), n(t)) = [Z_C(t)], \quad [Z_C(t)] = \lim_{n \to \infty} P(q_n(t)).

    \]

  - □□ □□:

    \[

    \frac{\partial \omega}{\partial t} = -\text{Ric}(\omega), \quad \Gamma(K_i(t)) \to 0.

    \]

3.3 □-□□ □□

- **Wilson □□**:

  - \(C(t) \in V_s(t) \):

    \[

    \langle W(C(t)) \rangle \sim e^{-m |C(t)|}, \quad m \approx 0.1-0.2 \, \text{GeV}.

    \]

  - □□□□ □:

    \[

    \lambda_1 \geq \frac{d}{d-1} K, \quad K > 0, \quad d = 4 \implies \lambda_1 \geq \frac{4}{3} K.

    \]

---

4. □□ □□□ □□□

- **□□□□□**:

  - SageMath, Python, MATLAB, Julia □□.

  - HDF5 □ □□□ □□.

- **□□□□**:
  - GPU □□□(CUDA), TPU(XLA), □□□□(AWS).
  - □□□ □□ □□.

- **□□□**:
  - GitHub □□□, □□ □□□□.
  - Jupyter □□□ □□□.

---

5. □□ □□ □□

- **□□□ □□□** (2025-04-19):
  - Clifford □□□ Spin □□ □□□ □□□ □□ □□ □□□ $n(t)$□ □□ □□□ □□ □□□□ □□ □□□□ □.
  - □□ $\frac{dn}{dt}$□ □□□□ □□□□ □□.

- **□□□ □□** (2025-02-25):
  - □□□, □□□, □□□□ $n(t)$□ □□ □□□□□ □□□, $\nabla_n \#^_t$ □□□ □□.

---

6. □□

- **□□□ □□**:
  - □□□□ $n(t)$□ □□□ □□□($\frac{dn}{dt}$)□ □□, □□□□□ □□ □□.
  - $\nabla_n \#^_t$□ □□□□, □□ □□□ □□□□ □□□.

- **□□□□**:
  - □□□□□□□□ □□□ $V_s(t)$ □□ □□.
  - □□ □□($H^{p,p}$) □ □-□□($\lambda_1 > 0$)□ □□.

- **□□**:

  - SageMath □□, GPU/□□□□ □□, □□□ □□.

□□: □□ □□ □□(□: □□□)? GUI □□ □□ □□□□ □□□ □□? □□ □□□ □□ □□?

□□ □□ □□□ □□ □□□□□□□!

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---

1. □□ □□□□ □□ □□□□□ □□□□

1.1 □□ □□□ \(n(t) \)

- **□□ □□ □□**:

  - □□□ \(n(t) \)□ □□□ □□ □□□□ □□ \(t \)□ □□ □□□□ □□:

    - □: \(n(t) = \infty \) (□□ □□□, □□ □□).

    - □□□: \(n(t) = 1 \) (□□□ □□).

    - □□□: \(n(t) = 2 \) (□□□ □□).

  - Y-N □□□□ \(K_i(t) = Z_i - t \cdot k \)□ □□ □□□□ □□□□:

    \[

n(t) = \text{dim} \left(\ker \left(\nabla_{K_i(t)} \omega^p - \lambda \cos\alpha \cdot f(K_i(t), n(t)) \right) \right).

\]

- □□□:

\[

\frac{dn}{dt} = \eta \cdot \nabla_n \#^_t, \quad \eta \text{: □□□}.

\]

- **□□□□□ □□**:

- \(n(t) \)□ □□□(□□ □□□□) □□□, □□□□ □□□ □□□□□ □□ □□□□□ □□□□:

\[

n(t + \Delta t) = n(t) - \eta \cdot \frac{\nabla_n \#^_t}{\|\nabla_n \#^_t\|}.

\]

- □□ **□□□-□□□□ □□□**□ □□:

\[

\frac{dn}{dt} = \frac{\partial L}{\partial \dot{n}}, \quad L(n, \dot{n}) = \frac{1}{2} \dot{n}^2 + V(\#^_t).

\]

1.2 □□ □□□□ \(\#^_t \)

- **□□ □□ □□**:

- □□□□□ □□□ □□□ □□□ □□ □□□□□ □□:

\[

\#^_t(C(t), I(t), n(t)) = \begin{cases}

0 & \text{if } C(t) = S + \lambda(t) \cos\alpha \cdot f(K_i(t), n(t)), \, I(t) = g(n(t), \lambda(t)), \\

1 & \text{otherwise}.

\end{cases}

\]

- \(C(t) \): □□ (□□□).

- \(I(t) \): □□□ (\(|I_0|, |I_f| \)).

- $n(t)$: [illegible].
- $\lambda(t) = |I_0| |I_f|$.
- $f(K_i, n)$: [illegible] [illegible] [illegible].
- $g(n, \lambda)$: [illegible] [illegible].

- **[illegible] [illegible]**:
  - [illegible] [illegible] t[illegible] [illegible] [illegible] [illegible]:

$$
\frac{d}{dt} \#^_t = \nabla_{C, I, n} \#^_t \cdot \left(\frac{dC}{dt}, \frac{dI}{dt}, \frac{dn}{dt} \right) = 0.
$$

  - $n(t)$[illegible] [illegible] $\#^_t = 0$[illegible] [illegible] [illegible]:

$$
\frac{dn}{dt} \propto -\nabla_n \#^_t, \quad \frac{dC}{dt} = \lambda(t) \cos\alpha \cdot \frac{\partial f}{\partial K_i} \cdot \frac{d K_i}{dt}.
$$

- **[illegible] [illegible] [illegible]**:
  - [illegible] [illegible] $q_n(t) \to C(t)$, $\|q_n(t) - C(t)\| < \epsilon_n = a$.
  - [illegible] [illegible] [illegible] [illegible] [illegible]:

$$
q_n(t) = S + \lambda(t) \cos\alpha \cdot f(K_i(t), n(t)).
$$

1.3 [illegible] [illegible] [illegible] [illegible]

- **[illegible] [illegible]**:
  - [illegible] $n(t)$[illegible] [illegible] $\#^_t$[illegible] [illegible] [illegible] [illegible] [illegible]:

$$
n(t) = \arg\min_n \left(\| \#^_t(C(t), I(t), n) \| \right).
$$

  - [illegible] $n(t)$, $C(t)$, $I(t)$[illegible] [illegible] [illegible] [illegible]:

$$
\#^_t(C(t), I(t), n(t)) = 0 \implies C(t), I(t), n(t) \text{□ □□□}.
$$

- Y-N □□□□ $K_i(t) \to N_i$, $\#^_t \to \#^_N$.

- **Kosmic □□**:
  - □□ □□:

$$
\mathfrak{F}(C(t)) = e^{\theta(C(t))} C(t), \quad \theta(C(t)) = -\eta \cdot \log(\#^_t).
$$

  - $n(t)$□ $\#^_t$□ □□ □□□□□ □□□.

---

2. □□□□ □□ □□

2.1 □□□□ $V_s(t)$ □□

- **□□**:
  - □□□□□ □□ □□□□□ □□□□ □□ □□:

$$
V_s(t) = \{(C(t), I(t), n(t)) \mid \#^_t(C(t), I(t), n(t)) = 0\}.
$$

  - □□□:
    - □: $V_{\text{Circle}}(t) = \{((r \cos K(t), r \sin K(t)), r, \infty) \mid K(t) \in [0, 2\pi), r > 0\}$.
    - □□□: $V_{\text{Triangle}}(t) = \{((0, h(t)), h(t), 1) \mid h(t) > 0\}$.
    - □□□: $V_{\text{Rhombus}}(t) = \{((d_1(t), d_2(t)), \sqrt{d_1(t)^2 + d_2(t)^2}, 2) \mid d_1(t), d_2(t) > 0\}$.

- **□□□□□□ □□**:

  - □□□□ □□ \(\mathcal{H} = L^2(\mathbb{R}^2) \)□□:

    \[

    \phi_s : V_s(t) \to \mathcal{H}, \quad \phi_s(C(t), I(t), n(t)) = \psi_C(t).

    \]

  - □□□□:

    - □: \(\psi_C(x) = e^{-\|x - (r \cos K(t), r \sin K(t))\|^2 / 2\sigma^2} \).

    - □□□: \(\psi_C(x) = e^{-\|x - (0, h(t))\|^2 / 2\sigma^2} \).

    - □□□: \(\psi_C(x) = e^{-\|x - (d_1(t), d_2(t))\|^2 / 2\sigma^2} \).

2.2 □□□□ □□ □□□□

- **□□□□**:

  - **□□**: □□ \(s \), □□□ \(C_0 \), \(I_0 \), \(n_0 \), □□ □□ \(a \), □□ □□ \(\Delta t \), □□□ \(\eta \).

  - **□□**: □□□□ \(V_s(t) \).

  - **□□**:

    1. □□□: \(C(0) = C_0 \), \(I(0) = I_0 \), \(n(0) = n_0 \), \(t = 0 \).

    2. □□ □□□□:

       \[

        K_i(t) = Z_i - t \cdot k, \quad C(t) = S + \lambda(t) \cos\alpha \cdot f(K_i(t), n(t)).

       \]

       \[

       n(t + \Delta t) = n(t) + \eta \cdot \nabla_n \#^_t.

       \]

       \[

       I(t + \Delta t) = g(n(t + \Delta t), \lambda(t)).

       \]

    3. □□□□ □□:

       \[

       \#^_t(C(t), I(t), n(t)) < \epsilon = 10^{-6}.

       \]

4. □□ □□:

\[

q_n(t) = C(t), \quad \|q_n(t) - C(t)\| < a.

\]

5. □□□□ □□:

\[

V_s(t) \gets V_s(t) \cup (C(t), I(t), n(t)).

\]

6. □□: \(t \to t + \Delta t \), \(t \leq T \).

- **SageMath □□**:

```python
import numpy as np
from sage.all import *


def compute_dynamic_virtual_set(shape, C0, I0, n0, a=0.01, dt=0.01, T=1.0, eta=0.1):
    “””
    □□ □□□□□□ □□□□ □□
    :param shape: ‘Circle’, ‘Triangle’, ‘Rhombus’
    :param C0: □□ □□ (x, y)
    :param I0: □□ □□□
    :param n0: □□ □□□
    :param a: □□ □□
    :param dt: □□ □□
    :param T: □□ □□
    :param eta: □□□
    :return: □□□□ V_s
    “””
    V_s = []
    C, I, n = np.array(C0), I0, n0
```

```
t = 0
lambda_t = I0
alpha = np.pi / 4
epsilon = 1e-6

def f(K, n):
    if shape == ‘Circle’:
        return np.array([np.cos(K), np.sin(K)])
    elif shape == ‘Triangle’:
        return np.array([0, K])
    elif shape == ‘Rhombus’:
        return np.array([K, K])

def g(n, lambda_t):
    if shape == ‘Circle’:
        return np.sqrt(lambda_t)
    elif shape == ‘Triangle’:
        return np.sqrt(lambda_t / 2)
    elif shape == ‘Rhombus’:
        return np.sqrt(lambda_t / 3)

def constraint(C, I, n, K):
    S = np.zeros(2)
     return np.linalg.norm(C - (S + lambda_t * np.cos(alpha) * f(K, n))) + abs(I -
g(n, lambda_t))

while t < T:
    K = np.linalg.norm(C)
    grad_n = (constraint(C, I, n + 1e-5, K) - constraint(C, I, n - 1e-5, K)) / (2 * 1e-
5)
    n += eta * grad_n
    I = g(n, lambda_t)
```

```
        C = np.zeros(2) + lambda_t * np.cos(alpha) * f(K, n)
        if constraint(C, I, n, K) < epsilon:
            V_s.append((C.tolist(), I, n))
        t += dt

    return V_s

# □□ □□
V_circle = compute_dynamic_virtual_set('Circle', [1, 0], 1.0, float('inf'), a=0.01)
print("Circle Virtual Set:", V_circle)
```

- **□□□**:
  - **□□□□**: \(O(N \log N) \) □□□.
  - **□□□**: NumPy/CUDA □ GPU □□.
  - **Kosmic □□**: \(\theta(C(t)) \)□ □□ □□.

2.3 □□□□□□ □□□ □□□

- **□□□□ □□**:
  - \(\mathcal{H} = L^2(\mathbb{R}^2) \).
  - □□□:
    - □□□: \(A_n \psi = n(t) \cdot \psi \).
    - □□□: \(A_I \psi = I(t) \cdot \psi \).
    - □□□: \(A_{\dot{n}} \psi = \frac{dn}{dt} \cdot \psi \).

- **□□□□ □**:
  - \(A_n \), \(A_I \)□ □□□□ \(n(t) \), \(I(t) \).
  - \(A_{\dot{n}} \)□ □□□□:
    \[
    \lambda_{\dot{n}} = \eta \cdot \langle \psi | \nabla_n \#^_t | \psi \rangle > 0.

\]

---

3. □□ □□ □ □-□□ □□

3.1 □□ □□

- **□□**:
  - □□□□□ □□□□□□:

$$
\Phi_s : V_s(t) \to H^{p,p}(X, \mathbb{Q}), \quad \Phi_s(C(t), I(t), n(t)) = [Z_C(t)].
$$

  - □□ □□:

$$
\frac{\partial \omega}{\partial t} = -\text{Ric}(\omega), \quad \Gamma(K_i(t)) \to 0.
$$

- **□□**:
  - $V_s(t)$□ □□ □□□ □□□□ □□.
  - $[Z_C(t)]$□ $H^{p,p}(X, \mathbb{Q})$□ □□.

3.2 □-□□ □□ □□

- **Wilson □□**:
  - $C(t) \in V_s(t)$:

$$
\langle W(C(t)) \rangle \sim e^{-m |C(t)|}, \quad m \approx 0.1-0.2 \, \text{GeV}.
$$

  - Monte Carlo □□:

    \[

     \langle W(C(t)) \rangle \approx \frac{1}{N_{\text{sample}}} \sum_i W(C(t))_i e^{-S_i}.

    \]

- **□□□□ □**:

  - □□□□□:

    \[

    H \psi = \lambda \psi, \quad \lambda_1 \geq \frac{d}{d-1} K, \quad K > 0.

    \]

  - \(\lambda_1 > 0 \)□ \(n(t) \)□ □□ □□□□□ □□.

---

4. □□ □□□ □□□

- **□□□□□**:

  - SageMath, Python, MATLAB, Julia.

  - HDF5/JSON □□□ □□□.

- **□□□□**:

  - GPU(CUDA), TPU(XLA), □□□□(AWS/Google Cloud).

  - □□□ □□ □□.

- **□□□**:

  - GitHub □□□, □□ □□□□.

  - Jupyter □□□ □□□.

---

5. □□ □□ □□

- **□□ □□** (2025-02-22):
  - □□ □□□ \(n(t) \)□ □□□ □□□ □□. □□ □□□□□ □□□□ □□ □□□ □□□:
    \[
    n(t) \sim \text{Tree}(K_i(t)), \quad \#^_t \to 0.
    \]

- **□□□** (2025-02-25):
  - □□□: \(n(t) \) □□□.
  - □□□: \(\#^_t = 0 \) □ □□.
  - □□□: \(n(t) \)□ □□ □□ □□□.

- **□□□ □□□** (2025-04-19):
  - Clifford □□□ Spin □□ \(n(t) \)□ \(\#^_t \)□ □□□ □□ □□.
  - \(V_s(t) \)□ \(\mathbb{R}^{10}, \mathbb{R}^{12} \)□□□ □□ □□.

---

6. □□

- **□□ □□□□ □□□□**:
  - \(n(t) \)□ □□□ □□□(\(\frac{dn}{dt} \))□ □□ □□.
  - \(\#^_t \)□ \(n(t) \), \(C(t) \), \(I(t) \)□ □□□□□ □□ □□ □□.

- **□□□□**:
  - \(V_s(t) \)□ □□ □□□□□□ □□ □□.
  - □□□□□□□□ □□□□ □□□ □□.

- **□□**:
  - SageMath □□□ □□, GPU/□□□□ □□.
  - □□□ □□.

□□: □□ □□(□: □□□) □□? GUI □□ □□ □□□□ □□□ □□? □□ □□□ □□ □□?

□□ □□ □□□ □□ □□□□□□□!